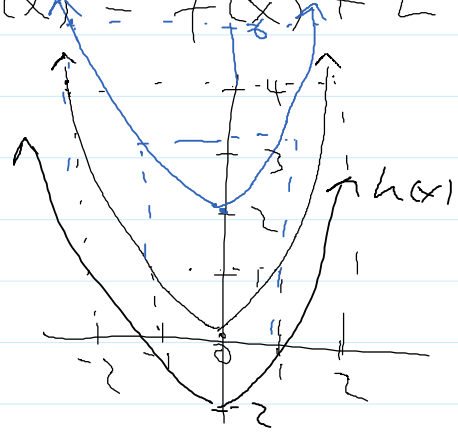
1.6 Transformation of Graphs

1. Vertical shift

Example

$f(x) = x^2$

$g(x) = f(x) + 2$ (vertical shift of $f(x)$ 2 units up)



$f(x) = x^2$

x	$f(x)$	$g(x)$
-2	$f(-2) = (-2)^2 = 4$	$f(-2)+2 = 4+2 = 6$
-1	$f(-1) = (-1)^2 = 1$	$f(-1)+2 = 1+2 = 3$
0	$f(0) = 0^2 = 0$	$f(0)+2 = 0+2 = 2$
1	$f(1) = 1^2 = 1$	$f(1)+2 = 1+2 = 3$
2	$f(2) = 2^2 = 4$	$f(2)+2 = 4+2 = 6$

Similarly,

$h(x) = f(x) - 2$ (vertical shift 2 units down)

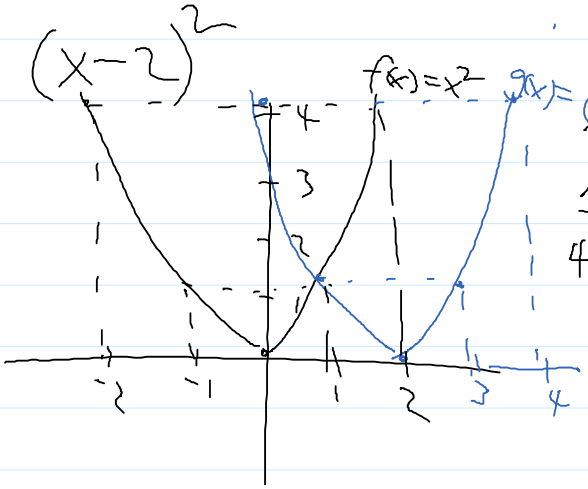
Horizontal shift

Example

$f(x) = x^2$

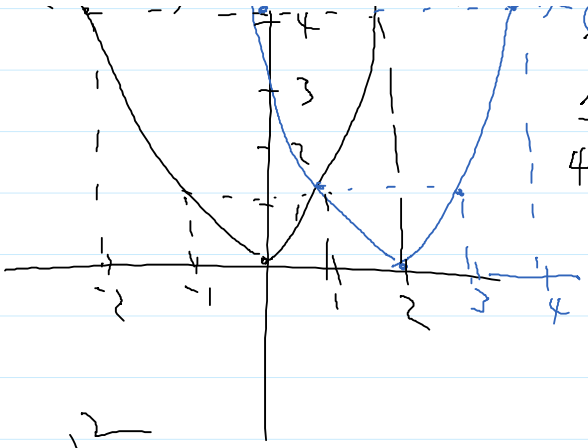
$g(x) = (x-2)^2$

(horizontal shift 2 units right)



x	$f(x) = x^2$	$g(x) = (x-2)^2$
-2	4	$(-2-2)^2 = 16$
-1	1	$(-1-2)^2 = 9$
0	0	$(0-2)^2 = 4$
1	1	$(1-2)^2 = 1$
2	4	$(2-2)^2 = 0$
3	9	$(3-2)^2 = 1$
4	16	$(4-2)^2 = 4$

(horizontal shift 2 units right



x	$(x-2)^2$
2	4
3	9
4	16

$(2-2)^2 = 0$
$(3-2)^2 = 1$
$(4-2)^2 = 4$

$h(x) = (x+2)^2$
horizontal shift
2 units left

Suppose you are given

$$f(x) = x^2$$

($g(x)$ is a transformation of $f(x)$)

$$g(x) = a(x-h)^2 + k$$

- a: if $a > 1$, vertical stretching; if $0 < a < 1$, vertical shrinking
- h: horizontal shift, right if $h > 0$, left if $h < 0$
- k: vertical shift, up if $k > 0$, down if $k < 0$

Identify the transformation

1. $f(x) = \sqrt{x}$, $g(x) = \frac{1}{2}\sqrt{x} - 2$

($\frac{1}{2}$: vertical shrinking; -2: vertical

Vertical shrinking

Vertical shift 2 units down

2. $f(x) = |x|$, $g(x) = -|x| - 4$

reflection about x-axis

vertical shift down 4 units

3. $f(x) = x^3$, $g(x) = -(x+1)^3 - 3$

reflection

horizontal shift left 1 unit

vertical shift down 3 units

4. $f(x) = x^2$, $g(x) = 2(x-3)^2 + 1$

vertical stretch

horizontal shift right 3 units

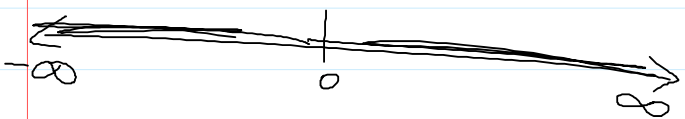
vertical shift up 1 unit

1.7 Combination of functions; Composite functions

Aside

How to find domain of functions

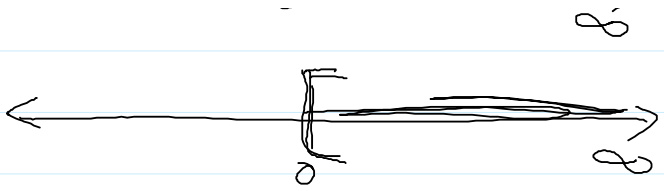
Number line



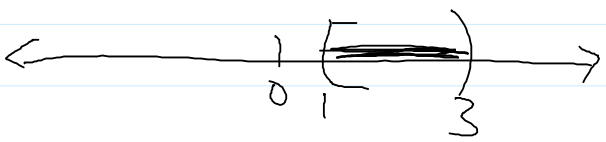
Interval Notation

$(-\infty, \infty)$

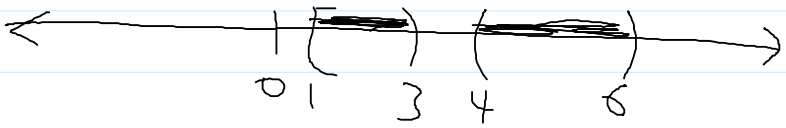
$[\quad , \infty)$



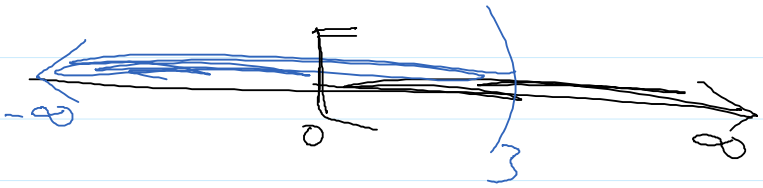
$[0, \infty)$



$[1, 3)$



$[1, 3) \cup (4, 6)$



$[0, 3)$

Suppose f, g is a function

and domain of f is $[0, \infty)$
domain of g is $(-\infty, 3)$

If I create a new function

$$(f+g)(x) = f(x) + g(x)$$

Q. What is the domain of $f+g$

A $[0, 3)$ (Intersection of Domain of f and g

1.7 Combination of functions

let f, g be functions

let Dom_f = Domain of f

Dom_g = Domain of g

1. $(f+g)(x) = f(x) + g(x)$, $\text{Dom}_{f+g} = \text{Dom}_f \cap \text{Dom}_g$

2. $(f-g)(x) = f(x) - g(x)$, $\text{Dom}_{f-g} = \text{Dom}_f \cap \text{Dom}_g$

3. $(fg)(x) = f(x) \cdot g(x)$, $\text{Dom}_{fg} = \text{Dom}_f \cap \text{Dom}_g$

4. $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, $\text{Dom}_{\frac{f}{g}} = \text{Dom}_f \cap \text{Dom}_g$, where $g(x) \neq 0$

Turn in on Monday
(show work)

1. Find the domain of

$$f(x) = \frac{1}{x-2} - \frac{1}{x+3}$$

2. $f(x) = \frac{3x+7}{x^2-16}$, $g(x) = \frac{5x-4}{x^2-16}$

find

(i) $f+g$
(ii) $f-g$
(iii) fg
(iv) $\frac{f}{g}$

(Determine the domain of each

3. Find the domain of

$$f(x) = \sqrt{30 - 3x}$$

Composition of function

let f, g be functions

$$(f \circ g)(x) = f(g(x))$$

$$(g \circ f)(x) = g(f(x))$$