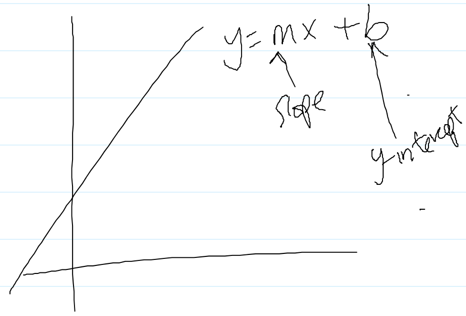
1.5 More on slopes

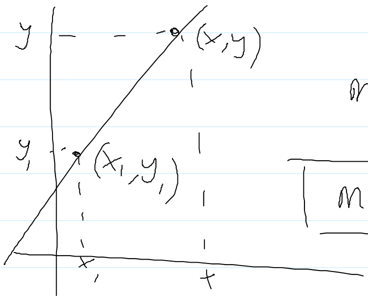
Recall 1.4

Linear functions



More on 1.4

Point slope form



$$m = \frac{y - y_1}{x - x_1}$$

$$\boxed{m(x - x_1) + y_1 = y}$$

Slope-intercept form

from the point slope form

$$y = m(x - x_1) + y_1$$
$$= mx - mx_1 + y_1 \qquad \text{Set } b = -mx_1 + y_1$$

$$\boxed{y = mx + b}$$

y-intercept (set $x = 0$)

$$y = m \cdot 0 + b$$
$$= b$$

x-intercept (set $y = 0$)

$$0 = mx + b$$
$$-b \quad -b$$
$$-b = mx$$
$$\frac{-b}{m} = x$$

general form

from slope intercept form

$$y = mx + b$$
$$y - mx - b = 0$$
$$\boxed{Ay + Bx + C = 0}$$

Exercise

Write the equation in general form $(Ay + Bx + C = 0)$

slope intercept form

$$\boxed{y = \frac{2}{3}x + 4}$$
$$-4$$

$$3\left(y - \frac{2}{3}x - 4\right) = 0 \cdot 3$$

$$\boxed{y = \frac{2}{3}x + 4}$$
$$\quad -4 \qquad\qquad -4$$
$$y - 4 = \frac{2}{3}x$$
$$-\frac{2}{3}x \qquad -\frac{2}{3}x$$
$$y - \frac{2}{3}x - 4 = 0$$

$$3\left(y - \frac{2}{3}x - 4\right) = 0 \cdot 3$$
$$\boxed{3y - 2x - 12 = 0}$$
general form

Exercise

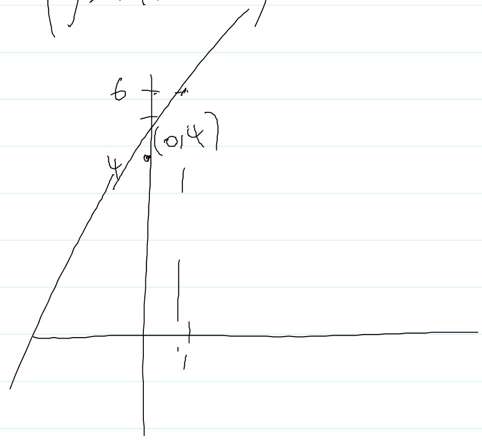
use the slope and y-intercept to graph the linear function

$$6x - 3y = -12$$

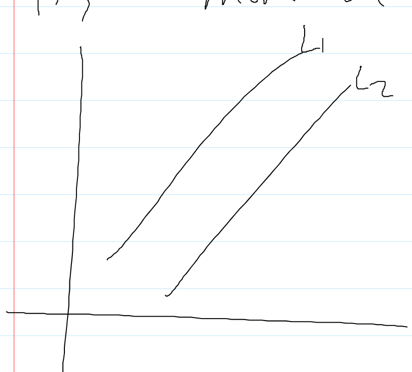
Hint

re-write in slope-intercept form $(y = mx + b)$

$$6x - 3y = -12$$
$$-6x \qquad\qquad -6x$$
$$\frac{-\cancel{3}y}{-\cancel{3}} = \frac{-6x - 12}{-3}$$
$$y = \frac{-\cancel{3}(2x + 4)}{-\cancel{3}}$$
$$\boxed{y = 2x + 4}$$



1.5 More on slopes



claim:

Let line L_1 and line L_2 be two non-vertical lines

If L_1 is parallel to L_2

then $m_1 = m_2$

where m_1 is slope of L_1

m_2 is slope of L_2

m_2 is slope of L_2

Exercise

write the equation of the line in slope-intercept form passing through $(-3,-1)$ and parallel to $y=-\frac{1}{2}x-5$

L_1: $(-3,-1)$ is on L_1 $\qquad$ L_1 is parallel to L_2

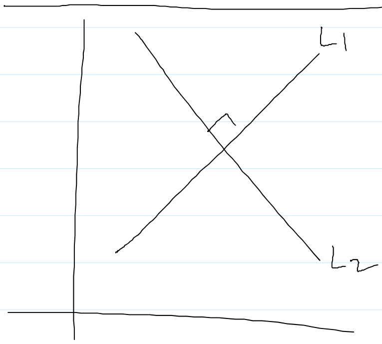
L_2: $y=-\frac{1}{2}x-5$

m is slope of L_1

$m=-\frac{1}{2}$, $(x_1,y_1)=(-3,-1)$

using point-slope form to find equation of L_1

$$m(x-x_1)+y_1=y$$
$$-\frac{1}{2}(x-(-3))+(-1)=y$$
$$-\frac{1}{2}(x+3)-1=y$$
$$-\frac{1}{2}x-\frac{3}{2}-1=y$$
$$-\frac{1}{2}x-\frac{5}{2}=y$$



If L_1 is perp to L_2

then $m_1 \cdot m_2 = -1$

m_1 = slope of L_1

m_2 = slope of L_2

$$m_1=\frac{-1}{m_2}$$

(m_1 is the negative reciprocal of m_2)

Exercise

Write the equation of the line in general form through $(-5,-3)$ and perp to $3x-y+4=0$

L_1: $(-5,-3)$ is on L_1

L_2: $3x-y+4=0$

$$-3x \qquad -3x$$

$$-y+4=-3x$$

$$-4 \qquad -4$$

$$\frac{+y}{+1}=\frac{-3x-4}{-1}$$

$$y=\frac{+(3x+4)}{+1}$$

$$\boxed{y=3x+4}$$

$$3x-y+4=0$$

$$+y \qquad +y$$

$$3x+4=y$$

slope of L_2 is 3

slope of L_1 is $-\frac{1}{3}$

$$3\cdot\left(-\frac{1}{3}\right)=-1$$

$$m_2\cdot m_1$$

$m=-\frac{1}{3}$, $(x_1,y_1)=(-5,-3)$

using point-slope form

$$m(x-x_1)+y_1=y$$

$$-\frac{1}{3}(x-(-5))+(-3)=y$$

$$-\frac{1}{3}(x+5)-3=y$$

$$-\frac{1}{3}x-\frac{5}{3}-3=y$$

$$3\left(-\frac{1}{3}x-\frac{14}{3}\right)=(y)3$$

$$-x-14=3y$$

$$3y+x+14=0$$

Exercise

The graph of f passes through $(2,-5)$ and is perpendicular to the line that

has an x-intercept of 4 and a y-intercept of -1.
Write the slope-intercept form of f

L_1: $(2,-5)$ $\qquad$ L_1 is perp L_2

L_2: x-intercept 4, y-intercept -1

$\overset{x_1\ y_1}{(4,0)}$ $\qquad$ $\overset{x_2\ y_2}{(0,-1)}$

$$\text{slope of } L_2 = \frac{y_2-y_1}{x_2-x_1} = \frac{-1-0}{0-4} = \frac{-1}{-4} = \frac{1}{4}$$

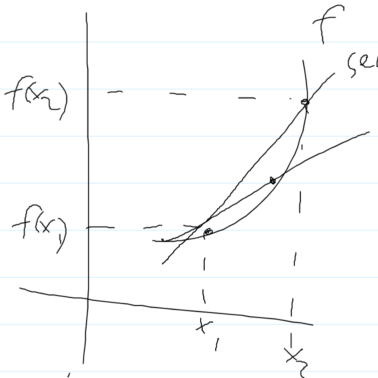
what is slope of L_1 : -4

$$m_1 \cdot m_2 = -1$$
$$(-4)\left(\frac{1}{4}\right) = -1$$

Using point slope form

$$m(x-x_1)+y_1 = y$$
$$-4(x-2)+(-5) = y$$
$$-4(x-2)-5 = y$$
$$-4x+8-5 = y$$
$$\boxed{-4x+3 = y}$$

Rate of change



Slope of the secant line = average rate of change between points x_1, x_2

Exercise

Find the average rate of change of

$f(x) = x^2 - x + 4$ from $x_1 = 2$ to $x_2 = 6$

avg rate of change $= \dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$

$= \dfrac{34 - 6}{6 - 2}$

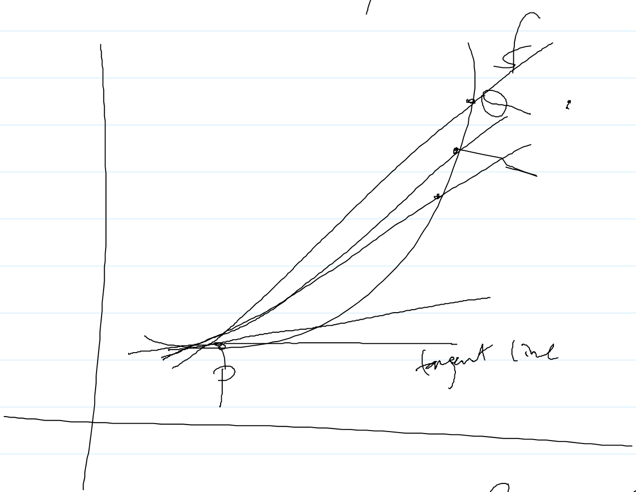
$= \dfrac{28}{4}$

$= 7$

$f(x) = x^2 - x + 4$

$f(x_2) = f(6)$
$= 6^2 - 6 + 4$
$= 36 - 6 + 4$
$= 34$

$f(x_1) = f(2)$
$= 2^2 - 2 + 4$
$= 4 - 2 + 4$
$= 6$



Tangent problem

Q. If you are given a point P on a graph

Can you write the equation of a line at P

as $Q \to P$, then slope secant line approaches the slope of the tangent line

1.6 Transformation of Graphs

Vertical shift

If $f(x)$ is a function, and c is a positive number

$f(x) + c$ is a vertical shift of $f(x)$ c units up

$f(x) - c$ is a vertical shift of $f(x)$ c units down

Horizontal shift

$f(x)$ is a function, c is positive number

$f(x-c)$ is an horizontal shift of $f(x)$
c units right

$f(x+c)$ is an horizontal shift of $f(x)$
c unit to the left

Vertical stretching and shrinking

$f(x)$ is a function, c is a positive number

1. If $c > 1$, $cf(x)$ is a vertical stretching of $f(x)$ by a factor of c
2. If $0 < c < 1$, $cf(x)$ is a vertical shrinking of $f(x)$ by a factor of c