Tues_04_19

Tuesday, April 19, 2022 2:35 PM

# Reminders

1. Take home exam given on Thursday 04/21 (due Tuesday 04/26)
2. Final Exam on May 03, 3:30 pm – 5:30 pm
3. Keep up with HW (many of you stopped doing your HW)
4. Tuesday 04/26 is the last day of class
5. Sections on Exam #3

   ~~10.1, 10.2~~ 11.1, 11.2, 11.3, 11.5, 12.1, 12.2, 13.1

   (13.2, 13.4 will be on the final).

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## 13.1 The time value of money

### 1. Simple Interest

Principal ($P$) (amount $)

Interest rate ($r$) (in %, convert to decimal)

time ($t$) (in years)

$$I = P \cdot r \cdot t$$

(Simple Interest)

$$A = P + I \quad \text{---} \quad (1)$$

($A$: Future value; $P$: present value)

Since $I = Prt$

(1) becomes

$$A = P + Prt$$

$$A = P(1 + rt)$$

**Exercise**

Find the simple interest owed for each loan

(a) $P = \$600$, $r$ at 2% for 1 year

$P = \$600$, $r = 0.02$, $t = 1$

$$I = P \cdot r \cdot t$$
$$= (600)(0.02)(1)$$
$$= \$12$$

(The Future value is

$A = P + I = \$600 + \$12 = \$612$)

$A = P(1 + rt)$

$A = P + I = \$600 + \$10 = \$610$

## Compound Interest

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$A$ ← Future value, $P$ ← present value

$r$ = interest rate (%, convert to decimal)

$t$ = time in years

$n$ = # of compounding in a year

annual compounding → $(n=1)$

bi-annual (semi-annual) ✓ → $(n=2)$

quarterly ✓ → $(n=4)$

monthly ✓ → $(n=12)$

⋮

daily ✓ → $(n=365)$

when $(n \to \infty)$, continuous compounding

$$A = Pe^{rt}$$

### Exercise

Find the future value when $P = \$4000$, $r = 2.5\%$, $t = 5$ years and interest is compounded annually

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$P = \$4000$, $r = 0.025$, $t = 5$, $n = 1$

$$A = 4000\left(1 + 0.025\right)^{5}$$

$$= \$4525.63$$

### More Exercise

Find the present value for the future value given

$A = \$1000$, $r = 3\%$, $t = 5$ years

(compounding is annually)

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$1000 = P\left(1 + 0.03\right)^{5}$$

$$\frac{1000}{1.03^5} = P$$

$$\$862.61 = P$$

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More Exercise

How long would it take to double your money in an account paying 4% compounded quarterly

(answer is in years, ignore leap years)

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$A = 2P$

$n = 4$ (quarterly compounding)

$r = 0.04$ (4% interest rate)

$$2P = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$2P = P\left(1 + \frac{0.04}{4}\right)^{4t}$$

$$2P = P\left(1.01\right)^{4t}$$

$$2 = 1.01^{4t}$$

(take log base 10 of both sides)

$$\log_{10} 2 = \log_{10} 1.01^{4t}$$

$$\frac{\log_{10} 2}{4 \log_{10} 1.01} = \frac{4t \log_{10} 1.01}{4 \log_{10} 1.01}$$

$$\frac{\log_{10} 10}{4 \log_{10} 1.01} \qquad \frac{\log_{10} 10}{4 \log_{10} 1.04}$$

$$17.415 \approx t$$

$t$ = 17 years and 152 days

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## Effective annual yield

$$Y = \left(1 + \frac{r}{n}\right)^n - 1$$

$r$ = nominal rate

$n$ = # of compounding

HW

1. Solve the effective annual yield formula for $r$

2. Ridgeway Savings compounds interest monthly. The effective annual yield is 1.95%. What is the nominal rate ($r$)