Tues_03_22

Tuesday, March 22, 2022 2:36 PM

Reminders

1. 10.3, 10.5 due Friday 03/25 11:59 pm
2. Exam #2 on 03/29

10.3 (Recall)

Permutation (n distinct objects taken r at a time) order is important

$$_nP_r = \frac{n!}{(n-r)!}$$

Combination (n distinct object taken r at a time) (order is not important)

$$_nC_r = \frac{n!}{(n-r)!\,r!}$$

Selection, committee

Exercise

1. In how many ways could 15 people be divided into 5 groups containing respectively 1, 2, 3, 4, 5, people

approach 1: $_{15}C_1 \cdot {_{14}C_2} \cdot {_{12}C_3} \cdot {_9C_4} \cdot {_5C_5} = 37,837,800$

approach 2: $_{15}C_5 \cdot {_{10}C_4} \cdot {_6C_3} \cdot {_3C_2} \cdot {_1C_1} = 37,837,800$

2. In how many ways could eight people be divided

into 2 groups of 3 people and a group of 2 people

$$\frac{8C3 \cdot 5C3 \cdot 2C2}{2!} = 280$$

(3) In how many ways could 15 people be divided into 5 groups of 3 people

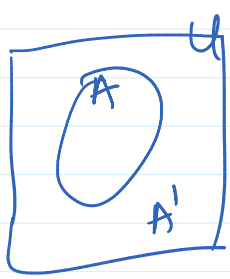
$$\frac{15C3 \cdot 12C3 \cdot 9C3 \cdot 6C3 \cdot 3C3}{5!}$$

$$15C3 = \frac{15!}{(15-3)!\,3!} = \frac{15!}{12!\,3!} = \frac{15 \cdot 14 \cdot 13 \cdot \cancel{12!}}{\cancel{12!}\,3!} = 5 \cdot 7 \cdot 13$$

$$= \frac{15 \cdot 14 \cdot 13}{3 \cdot 2}$$

10.5 Counting problems involving "Not" and "or"

Recall from Set Theory



$$n(A) + n(A') = n(U)$$

$$n(A) = n(U) - n(A')$$

$$n(A') = n(U) - n(A)$$

Example

(1) If you toss seven fair coins, in how many ways can you obtain each result

(a) at least one head

A = at least one head

A = at least one head

A' = None

$n(u) = \cancel{2222222} = 2^7$

$n(A) = n(u) - n(A')$
$= 2^7 - 1$
$= 128 - 1 = 127$

(b) A = at least 2 heads

A' = one head or none

$n(A) = n(u) - n(A')$
$= 2^7 - (7+1) = 120$

(c) A = at least 2 tails

A' = one tail or none

$n(A) = n(u) - n(A')$
$= 2^7 - (7+1)$
$= 120$

Exercise

(1) How many different ways could 3 distinct days of the week be chosen so that at least one of them begins with the letter 'S'
(assume order of selection is not important)

A = distinct choice of 3 days so that at least one of them begins with 'S'

A' = distinct choice of 3 days so that None begins with 'S'

$n(A) = n(u) - n(A')$
$= 7C3 - 5C3$

(2) Find the # of four-digit counting numbers containing at least one zero, under each condition

(a) repeated digits are allowed

A' = four digit # containing no zero

$\{0,1,2,3,4,5,6,7,8,9\}$

$n(A) = n(u) - n(A')$

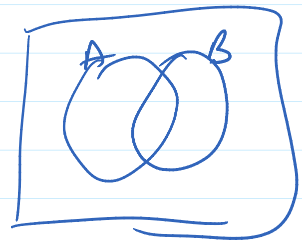
$9 \cdot 10 \cdot 10 \cdot 10 - 9 \cdot 9 \cdot 9 \cdot 9 = 9 \cdot 10^3 - 9^4 = 2439$

(b) repeated digit are not allowed

$$n(A) = n(U) - n(A')$$

$$= 9.9.8.7 - 9.8.7.6 = 1512$$

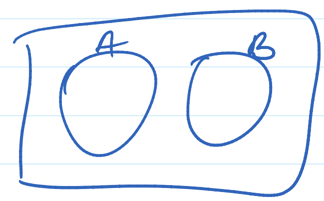
Problems involving 'or'



$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

If A, B are disjoint $(n(A \cap B) = \phi)$

$$n(A \cup B) = n(A) + n(B)$$



Exercise

Using the table below

	Men (M)	Women (W)	Total
(D) Democrat	8	4	12
(R) Republican	3	5	8
	11	9	20

(b) a man or a Democrat $n(M \cup D) = n(M) + n(D) - n(M \cap D)$

$11 + 12 - 8 = 15$

(c) a man or a Republican woman $n(M \cup RW) = n(M) + n(RW)$

$11 + 5 = 16$

(d) A woman or a Democrat man $n(W \cup DM) = n(W) + n(DM)$

$= 9 + 8 = 17$

Take home Exercise

52 cards in a deck

4 Suits (Club, hearts, Diamond, Spade) A 2 3 4 5 6 7 8 9 J Q K
(13) (13) (13) (13)

3 faced cards in each suits (12 faced cards).

Question
How many possible 5 cards are there in a poker hand from a 52 card deck $52C5 = 2598960$

Among the 2598960 possible 5-card poker hands from a 52-card deck how many contain the following cards

1. at least one card that is not a heart
2. Cards of more than one suit
3. at least one face card
4. at least one club, but not all clubs