

## Reminders

1. Grades (see D2L)
  2. Class Note (see D2L)
  3. Exam #2 Tuesday 03/29  
3.1, 3.2, 3.3, 3.4, 10.1, 10.2, 10.3, 10.5
  4. Study guide for Exam #2
  5. Office Hours (see Syllabus TR, 10-12 noon  
Ub RM 402  
Liverpool FC)
- Visit the  
Tutoring Lab
6. Mid-semester write up due tonight 11:59 pm
  7. HW 10.1, 10.2 due 03/8

## 10.2 The Fundamental Counting principle

### Uniformity Criterion

In a multiple part task

If the # of choices of each part does not depend on the # of choices of the other parts selected previously

Suppose we have a 3 part task

$$\boxed{9} \cdot \boxed{10} \cdot \boxed{110} = 9 \times 10^2 \quad \left( \begin{array}{l} \text{sample how many} \\ \rightarrow \dots \dots \dots \dots \dots \end{array} \right)$$

first digit      second digit      third digit = 900      '6 digit number is possible?  
 $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

## Fundamental Counting principle (FCP)

When a task consists of  $k$  separate parts and satisfies the uniformity criterion, if the first part can be done in  $n_1$  ways, the second part in  $n_2$  ways, ..., the  $k$ th part in  $n_k$  ways. The total number of ways to complete the task is

$$n_1 \cdot n_2 \cdot \dots \cdot n_k$$

### Exercise

1. How many ways can you arrange 3 people in a line

$$\begin{array}{ccc}
 \boxed{3} & \cdot & \boxed{2} & \cdot & \boxed{1} & = & 6 \text{ ways} \\
 \text{1st} & & \text{2nd} & & \text{3rd} & & \\
 \text{position} & & \text{position} & & \text{position} & & 
 \end{array}$$

2. How many 3 letters can be formed from the set  $\{A, B, C\}$

$$\begin{array}{ccc}
 \boxed{3} & \cdot & \boxed{3} & \cdot & \boxed{3} & = & 3 \cdot 3 \cdot 3 = 3^3 = 27 \\
 \text{first} & & \text{second} & & \text{third} & & \\
 \text{letter} & & \text{letter} & & \text{letter} & & 
 \end{array}$$

3. How many ways can you arrange 3 people in a line

Tom, Peter, Andrew

(given that Andrew must be first in the line)

$$\begin{array}{ccc}
 \boxed{1} & \cdot & \boxed{2} & \cdot & \boxed{1} & = & 2 \text{ ways} \\
 \text{first} & & \text{second} & & \text{third} & & \\
 \text{position} & & \text{position} & & \text{position} & & 
 \end{array}$$

(5) Selecting Dinner ~~the~~ items

- 5 choices on Soup & Salad category

(2 Soups, 3 Salad)

- 2 choices in bread category

- 4 choices in Entrée category

- (a) Find the number of dinners available  
one item should be included from each of the 3 categories

$$\boxed{5} \cdot \boxed{2} \cdot \boxed{4} = 40$$

Soup, Salad   Bread   Entrée

- (b) Only Salad and Entrée are to be included

$$\boxed{3} \cdot \boxed{4} = 12$$

Salad   Entrée

- (c) one Soup, one Salad, one Entrée

$$\boxed{2} \cdot \boxed{3} \cdot \boxed{4} = 24$$

Soup   Salad   Entrée

### More Exercise

1. How many nonrepeating odd 3-digit counting numbers are there

$\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$\{1, 3, 5, 7, 9\}$

$$\boxed{8} \cdot \boxed{8} \cdot \boxed{5}$$

first digit   second digit   third digit

- (2) Create a four-digit number from the set

$\{1, 2, 3\}$

How many of these numbers are odd and less than 2000

$$\boxed{1} \cdot \boxed{3} \cdot \boxed{3} \cdot \boxed{2} = 18$$

first digit   second digit   third digit   fourth digit

### Factorial

# Factorial

Consider the example below

How many ways can you line up 5 friends for photographs

$$\begin{array}{ccccccccc} \boxed{5} & \boxed{4} & \boxed{3} & \boxed{2} & \boxed{1} & = & 5! & = & 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \\ \text{1st} & \text{2nd} & \text{3rd} & \text{4th} & \text{5th} & & & & = 120 \end{array}$$

The arrangement of  $n$  objects

The # of ways to arrange  $n$  distinct objects is  $n!$

$$n! = n \cdot (n-1) \cdot (n-2) \cdot \dots \cdot 2 \cdot 1$$

$$0! = 1$$

$$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$$

$$1! = 1$$

## Exercises

$$\textcircled{1} \frac{9!}{7!(9-7)!} = \frac{9!}{7! \cdot 2!} = \frac{9 \cdot 8 \cdot \cancel{7!}}{\cancel{7!} \cdot 2!} = \frac{9 \cdot 8}{2 \cdot 1} = \frac{9 \cdot 8}{2} = 36$$

## Distinguishable Arrangements

### Example

How many ways can you arrange the letters in the following words

$$1. \text{ BANANA} = \frac{6!}{3! \cdot 2!} = \frac{6 \cdot 5 \cdot 4 \cdot \cancel{3!}}{\cancel{3!} \cdot 2!} = 60$$

## Home work

Raj keeps phone numbers for seven closest friends (3 men & 4 women)

in his digital phone memory. How many ways can he list them for the following conditions.

(a) men listed before women

(b) men are <sup>all</sup> listed together