

Reminders

1. HW 2.3, 2.4 due on 02/11, 11:59 PM
2. Exam #1 on 02/15
(see study guide)

De Morgan's Law (Continuation of 2.3)

$$1. A' \cup B' = (A \cap B)'$$

$$2. A' \cap B' = (A \cup B)'$$

We will use Venn diagrams to verify De Morgan's law

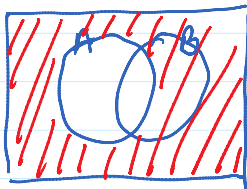
Prove that

$$A' \cup B' = (A \cap B)'$$

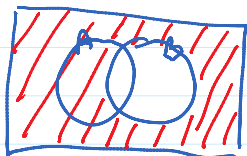
using Venn diagrams

PF

L.H.S



A' is shaded
 $A' = \{x \mid x \in U \text{ and } x \notin A\}$



B' is shaded
 $B' = \{x \mid x \in U \text{ and } x \notin B\}$

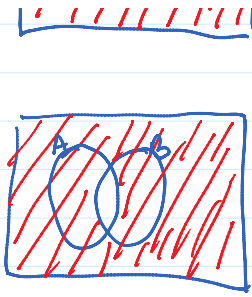
\dots

R.H.S



$(A \cap B)'$

$$(A \cap B)' = \{x \mid x \in U \text{ and } x \notin (A \cap B)\}$$



$A' \cup B'$
 Take union of
 A' and B' above
 $A' \cup B' = \{x \mid x \notin A \text{ or } x \notin B\}$

So L.H.S = R.H.S

Let us prove the second half of De Morgan's law

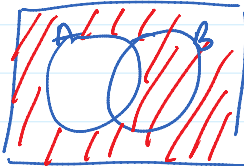
prove that

$$A' \cap B' = (A \cup B)'$$

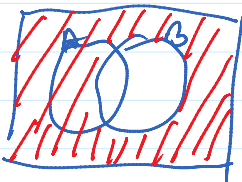
using Venn diagrams

pf

L.H.S



A' is shaded



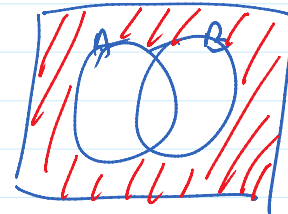
B' is shaded



$A' \cap B'$ is shaded

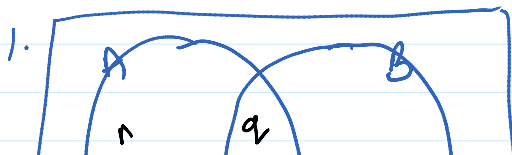
L.H.S = R.H.S

R.H.S



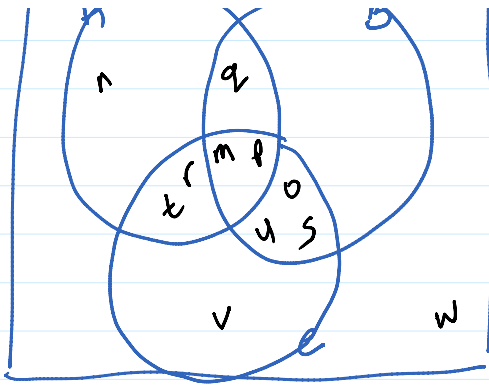
$(A \cup B)'$ is shaded

Exercise on Venn diagram



$$U = \{m, n, o, p, q, r, s, t, u, v, w\}$$

$$A = \{m, n, p, q, r, s\}$$



$$A = \{n, r, p, q, v, w\}$$

$$B = \{m, p, r, q, s, v\}$$

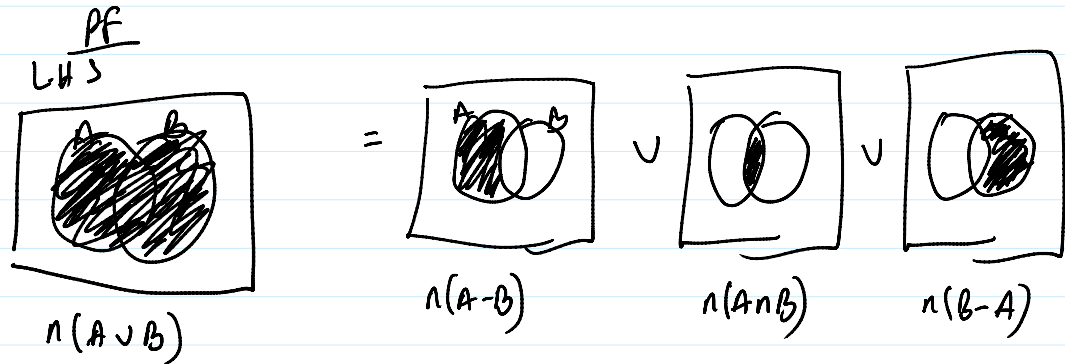
$$C = \{n, p, r, s, t, v, w\}$$

2.4 Surveys and Cardinal Numbers

Remark 1

Let A, B be sets

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$



So

$$n(A \cup B) = n(A-B) + n(A \cap B) + n(B-A) \quad (1)$$

Now,

$$n(A) = n(A-B) + n(A \cap B)$$

$$n(A) - n(A \cap B) = n(A-B) \quad (2)$$

Similarly for

$$n(B) = n(B-A) + n(A \cap B)$$

$$n(B) - n(A \cap B) = n(B-A) \quad (3)$$

Let us substitute (2) and (3) into (1)

$$\begin{aligned}n(A \cup B) &= n(A) - n(A \cap B) + n(A \cap B) + n(B) - n(A \cap B) \\ &= n(A) + n(B) - n(A \cap B)\end{aligned}$$

R.H.S

Exercise

1. Find the value of $n(A \cup B)$ if
 $n(A) = 12$, $n(B) = 14$, $n(A \cap B) = 5$

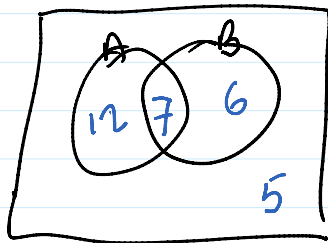
Hint

use $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$$\begin{aligned}&= 12 + 14 - 5 \\ &= 26 - 5 \\ &= 21\end{aligned}$$

2. Draw a Venn diagram and use the information to fill in the number of elements in each region

(9) $n(A) = 19$ $n(B) = 13$ $n(A \cup B) = 25$
 $n(A') = 11$



$$\begin{aligned}n(A \cup B) &= n(A) + n(B) - n(A \cap B) \\ 25 &= 19 + 13 - n(A \cap B) \\ 25 &= 32 - n(A \cap B)\end{aligned}$$

⇓

$$n(A \cap B) = 32 - 25$$

$$= 7$$

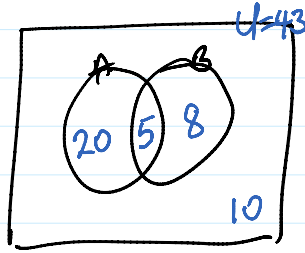
(b)

$$n(U) = 43$$

$$n(A) = 25$$

$$n(A \cap B) = 5$$

$$n(B) = 30$$



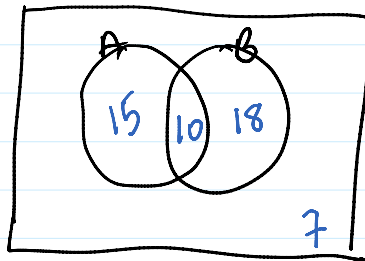
(c)

$$n(A') = 25$$

$$n(B) = 28$$

$$n(A' \cup B') = 40$$

$$n(A \cap B) = 10$$

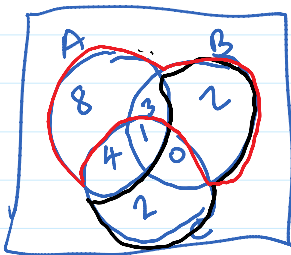


think
use De Morgan's law

$$n(A' \cup B') = n(A \cap B)'$$

Exercise

The cardinality of each region is given in the Venn diagram ~~the~~ below



A', C'

$$(a) n(A \cap B \cap C) = 1$$

$$(b) n(A \cap B' \cap C) = 4$$

$$(c) n(A' \cap B' \cap C) = 2$$

$$(d) n(A' \cap B \cap C') = 2$$

<u>Exercise</u>		Facilities			Totals
		B	R	O	
Personnel	C	12	29	54	95
	W	4	5	6	15
	\bar{E}	374	71	285	730
Totals		390	105	345	840

$$n(C) = 95$$

$$n(W) = 15$$

$$n(E) = 730$$

$$n(B) = 390$$

$$n(R) = 105$$

$$n(O) = 345$$

$$n(C \cap B) = 12$$

$$\begin{aligned} n(C \cup B) &= n(C) + n(B) - n(C \cap B) \\ &= 95 + 390 - 12 \\ &= 473 \end{aligned}$$

Find the following

1. $n(W \cap O)$
2. $n(R' \cup W')$
3. $n((C \cap B) \cup (E \cap O))$
4. $n((C \cup W) \cap (B \cup R))$
5. $n(B \cap (W \cup R)')$