Tips for learning advanced mathematics Carl Mummert

1. Know all the definitions and their negations - cold.

Why? You can't prove anything without knowing the definitions. When you want to use contraposition or contradiction, you need to know the negations. *Faulty negations are one of the most common problems students encounter when learning how to write proofs - and they are completely avoidable.*

2. Make a summary sheet of all theorems, lemmas, and examples covered in lecture or in the textbook.

Why? You need to gain the ability to write "routine" proofs quickly, and this requires having a good knowledge of the theorems so that you can see which ones apply to a certain situation. Rewriting all the theorems helps you learn their hypotheses and gives you a short reference sheet to use when studying.

3. If a theorem has a name, you should be able to state it immediately from memory if someone tells you the name.

Why? Results like "Fatou's lemma" or "Lagrange's theorem" are given names because they are referred to very often. You will find it hard to read or hear proofs that use these names if you don't know what the names refer to. At the same time, if a textbook or lecture has a theorem that does not have a name, the theorem must be important for some reason - figure out why while you are studying.

4. Try to identify common proof techniques the can be used in multiple proofs, and give them names.

Why? To write a proof, you have to be able to select the right technique. By giving names to common techniques, you help yourself remember them when you need them. For example, there are many "epsilon over two" arguments in analysis, and in measure theory there is a class of counterexamples that I call "Space Invaders". Use names that have a personal meaning for you.

5. Be very familiar with the proofs that are shown to you. Write them out on a board like you are lecturing, then repeatedly read through them aloud or present them to fellow students.

Why? In advanced math classes, we present proofs both to show you why things are true and to teach you techniques that you can apply to other problems. Your goal should be to completely understand these proofs. You should make it a goal to be able to write out a proof of any of the theorems on your summary sheet from memory without using any reference other than the summary sheet.