Thurs_03_17

Thursday, March 17, 2022 2:28 PM

Reminders

1. HW 10.1, 10.2 due Friday 03/18, 11:59 pm
2. Exam #2 03/29
3. See Exam #2 Study guide on D2L (click on the link under class note)
4. Project

Unfinished problem from last time

Home work

Raj keeps phone numbers for seven closest friends (3 men & 4 women) in his digital phone memory. How many ways can he list them for the following conditions.

(a) men listed before women

Men: 3 · 2 · 1 (1st, 2nd, 3rd) · Women: 4 · 3 · 2 · 1 (4th, 5th, 6th, 7th)

(b) men are all listed together

$3! \cdot 4! = 144$

(b) Since the men must be together, it is like arranging 5 phone numbers

So $5!$

But you still have to find an arrangement for the men $= 3!$

The complete solution becomes

$5! \cdot 3! = 720$ ways

10.3 Permutation and Combination

Permutation

You want to arrange n distinct objects taking r at a time $(r \leq n)$

$$nPr = \frac{n!}{(n-r)!}$$

Example

1. How many different ways could first, second, third place finishes occur in a race with 6 runners

approach 1 $\{A, B, C, D, E, F\}$

$$6P3 = \frac{6!}{(6-3)!} = \frac{6!}{3!} = \frac{6 \cdot 5 \cdot 4 \cdot \cancel{3!}}{\cancel{3!}} = 6 \cdot 5 \cdot 4 = 120$$

approach 2

$$\boxed{6} \cdot \boxed{5} \cdot \boxed{4} = 120$$

1st 2nd 3rd

(3) Identification # in a project consist of 3 letters followed by 3 digits and then 3 more letters

example(ABC123DEF)

Assume repitions are not allowed within any of the 3 groups, but letters in the first group ~~can~~ may occur in the last group. How many distinct identification # are possible ?

first 3 letters	3 digits	last 3 letters
Part1	Part2	Part3
$26P3$	$10P3$	$26P3$

$$= 1.75 \times 10^{11}$$

$$= 175\,219\,200\,000$$

Exercise

Radio Station in US have Call letters that begin with K or W. Some have 3 Call letters such as WBZ in Boston, WLS in Chicago and KGO in San Francisco. Assuming no repetition of letters, how many 3 letter set of call letters are possible?

Solution

{K, W} Part 1	Part 2 (choose 2 from 25 letters (since no repetition is allowed))
choose 1 from either K or W (2 choices)	$25P2$

$$= 2 \cdot (25P2) = 1200$$

Combination (watch for keywords 'selection', 'committee', ...)

Example

Consider the example of selecting 2 members from $\{A, B, C\}$

(a) order is important

$\{AB, BA, AC, CA, BC, CB\}$ $\quad 3P2 = \frac{3!}{(3-2)!} = \frac{3!}{1!} = \frac{3 \cdot 2 \cdot 1!}{1!} = 6$

(b) order is not important

$\{AB, AC, BC\}$ $\quad 3C2 = \frac{3!}{(3-2)!\,2!} = \frac{3!}{1!\,2!} = \frac{3 \cdot 2!}{1!\,2!} = 3$

Combination

# of subsets of n distinct things taken r at a time $(r \leq n)$

$$nCr = \frac{n!}{(n-r)!\,r!} = \frac{n!}{(n-r)!} \cdot \frac{1}{r!} = nPr \cdot \frac{1}{r!}$$

$$\boxed{nCr = nPr \cdot \frac{1}{r!}}$$

Exercise

1. How many ways can a sample of five cell-phones be selected from a shipment of 24 cell-phones

$$24C5 = \frac{24!}{(24-5)!\,5!} = 42504$$

2. In Superlotto plus, you select five distinct numbers from 1 to 47 and one Mega number from 1 to 27, hoping that your selection will match a random list selected by lottery officials

(a) How many different sets of six numbers can you select

Select 1-47 part 1	Mega select 1-27 part 2	
$47C5$	$27C1$	$= 41416353$

(b) Paul always includes his age and his wife's age as two of the first five numbers in his Superlotto plus selection. How many ways can he complete his list of six numbers.

part 1	part 2	
$47C3$	$27C1$	$= 437805$

Cards

A deck of Card (52 playing cards)

4 Suits } Spade, Hearts, Diamond, Clubs }
(13) (13) (13) (13)

A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K

26 Red Cards } Hearts, Diamonds }

26 Black cards } Spade, Clubs }

Exercise

How many different 5-card poker hands would contain only cards of a single suit?

$$4 \cdot {}_{13}C_5 = 5148$$