Tues_02_22

Tuesday, February 22, 2022 12:36 PM

Conditional Statements and Circuits

Recall the following line from

Field of Dreams

"If you build it, he will come"

p = you build it, q = he will come

Conditional Statement

$p \to q$

(p implies q)

If p, then q
If p, q
q if p

$$\underset{\text{antecedent}}{p} \to \underset{\text{consequent}}{q}$$

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

1. If the antecedent is false, then $p \to q$ is true
2. If the consequent is True, then $p \to q$ is true
3. $p \to q$ is false only when antecedent is true and consequent is false

Construct truth table

① $(p \wedge q) \to (p \vee q)$

p	q	$(p \wedge q)$	$(p \vee q)$	$(p \wedge q) \to (p \vee q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T

Tautology

T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

Tautology

② $(\sim p \to \sim q) \to (p \wedge q)$

p	q	$\sim p$	$\sim q$	$\sim p \to \sim q$	$p \wedge q$	$(\sim p \to \sim q) \to (p \wedge q)$
T	T	F	F	T	T	T
T	F	F	T	T	F	F
F	T	T	F	F	F	T
F	F	T	T	T	F	F

Thurs 02/24 (3.3 continued)

Reminders

1. HW 3.2 due Friday 02/25 11:59 pm
2. Exam 2 on Tuesday 03/29
3. Mid-Semester Write up due 03/15 11:59 pm
4. Mid-Semester Survey due 03/04
5. Exam 2 Study guide (see class page through D2L)
6. Sections on Exam 2 (3.1, 3.2, 3.3, 3.4, 10.1, 10.2, 10.3, 10.5)

More examples

Draw a truth table

① $[(r \vee p) \wedge \sim q] \to p$

antecedent

Consequent

Remark

If there are n component statements then there are 2^n rows in the truth table

(c) $[(r \vee p) \wedge \sim q] \rightarrow p$ | truth table

consequent

antecedent

p	q	r	$\sim q$	$(r \vee p)$	$(r \vee p) \wedge \sim q$	$[(r \vee p) \wedge \sim q] \rightarrow p$
T	T	T	F	T	F	T
T	T	F	F	T	F	T
T	F	T	T	T	T	T
T	F	F	T	T	T	T
F	T	T	F	T	F	T
F	T	F	F	F	F	T
F	F	T	T	T	T	F
F	F	F	T	F	F	T

$n = 3$

$2^n = 2^3 = 8$

Draw a truth table for the following

$p \rightarrow q$, $\sim p \vee q$, $\sim(p \rightarrow q)$, $p \wedge \sim q$

p	q	$\sim p$	$\sim q$	$p \rightarrow q$	$\sim p \vee q$	$\sim(p \rightarrow q)$	$p \wedge \sim q$
T	T	F	F	T	T	F	F
T	F	F	T	F	F	T	T
F	T	T	F	T	T	F	F
F	F	T	T	T	T	F	F

$\boxed{p \rightarrow q \equiv \sim p \vee q}$

$\sim(p \rightarrow q) \equiv p \wedge \sim q$

Draw a truth table for the following

$p \rightarrow q$, $q \rightarrow p$, $\sim p \rightarrow \sim q$, $\sim q \rightarrow \sim p$

p	q	$\sim p$	$\sim q$	$p \rightarrow q$	$q \rightarrow p$	$\sim p \rightarrow \sim q$	$\sim q \rightarrow \sim p$
T	T	F	F	T	T	T	T
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	T	T	T

$p \rightarrow q \equiv \sim q \rightarrow \sim p$

$q \rightarrow p \equiv \sim p \rightarrow \sim q$

Circuits

closed = true

Circuits

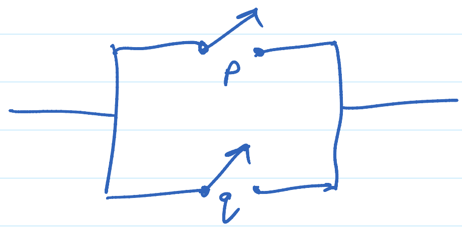
closed = true
open = false

Series Circuit

equivalent to $p \wedge q$

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

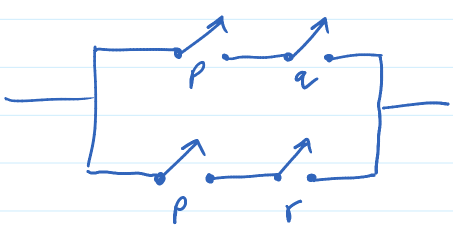
Parallel Circuit



equivalent to $p \vee q$

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

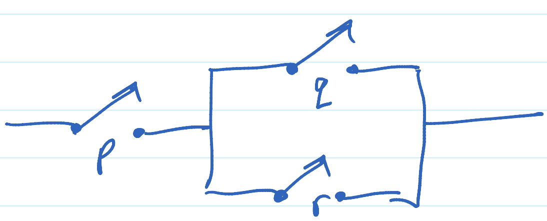
Consider the circuit below



$$(p \wedge q) \vee (p \wedge r)$$

You can verify that

$$(p \wedge q) \vee (p \wedge r) \equiv p \wedge (q \vee r)$$



Use the fact

$$p \to q \equiv \sim p \vee q$$

Draw a circuit for $p \to (q \wedge \sim r)$