Thurs_02_17

Thursday, February 17, 2022 2:53 PM

Reminders

1. hw 3.1 due Friday 02/18, 11:59 pm
2. Start working on the mid semester write up.

3.2 Truth Tables and Equivalent Statements

The Conjunction

Suppose p, q are statements

$p \wedge q$ - This is a compound statement

(This is true only when both p, q are true)

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Example

Let p = "$5 > 3$"

q = "$6 < 0$"

find the truth value of $p \wedge q$ ($5 > 3$ and $6 < 0$)

p is true

q is false

So $P \wedge Q$ is false

Disjunction

Let P, Q be Statements

$P \vee Q$ is false only if both P, Q are false

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Example

$P =$ "$5 > 3$" True

$Q =$ "$6 < 0$" False

$P \vee Q$ is true $5 > 3$ or $6 < 0$ is true

Negation

Let P be a statement
$\sim P$ is the negation of P

P	$\sim P$
T	F
F	T

Exercise

(1) If q is false, what must be the truth value of

(a) $(p \wedge \sim q) \wedge q$ (False)

$(p \wedge \sim F) \wedge F$

$(p \wedge T) \wedge F$

when $p = T$, $(p \wedge T) \wedge F = (T \wedge T) \wedge F$
$= T \wedge F$
$= F$

when $p = F$, $(p \wedge T) \wedge F = (F \wedge T) \wedge F$
$F \wedge F$
$= F$

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

p	$\sim p$
T	F
F	T

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(3) If $p \vee (q \wedge \sim q)$ is true, what must be the truth value of p (true)

$p \vee (q \wedge \sim q) = T$

when $q = T$, $q \wedge \sim q = T \wedge \sim T = T \wedge F = F$

$p \vee (q \wedge \sim q) = p \vee F = T$ (p must be true)

when $q = F$, $q \wedge \sim q = F \wedge \sim F = F \wedge T = F$

$p \vee (q \wedge \sim q) = p \vee F = T$ (p must be true)

Constructing Truth tables

(a) Draw truth table for $(q \vee \sim p) \vee \sim q$

(a) Draw truth table for $(q \vee \sim p) \vee \sim q$

p	q	$\sim p$	$\sim q$	$q \vee \sim p$	$(q \vee \sim p) \vee \sim q$
		F	F	T	T
		F	T	F	T
		T	F	T	T
		T	T	T	T

p	q	$(q \vee \sim p) \vee \sim q$					
T	T	T	T	F	T	T	F
T	F	F	F	F	F	T	T
F	T	T	T	T	T	T	F
F	F	F	T	T	T	T	T

(2) $\sim p \wedge q$

p	q	$\sim p$	$\sim p \wedge q$
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	F

p	q	$\sim p$	$\wedge$	q
T	T	F	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	F	F

Remark

A logical statement have n component statements will have 2^n rows in its truth table

Example

Construct truth table for

(1) $r \vee (p \wedge \sim q)$

(a) $r \vee (p \wedge \sim q)$

p	q	r	$\sim q$	$p \wedge \sim q$	$r \vee (p \wedge \sim q)$
T	T	T	F	F	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	F	F	T
F	T	F	F	F	F
F	F	T	T	F	T
F	F	F	T	F	F

Home work

Let p be a true statement and q, r be false statements find the truth value of the following compound statement

(a) $(p \wedge r) \vee \sim q = T$

(b) $p \wedge (q \vee r) = F$

(c) $\sim(p \wedge q) \wedge (r \vee \sim q) = T$

(d) $\sim[\sim q \vee (r \wedge \sim p)] = F$

(e) $\sim p \vee \sim q = \sim T \vee \sim F$

$= F \vee T$

$= T$

Tuesday 02/22 (Section 3.2 continued)

Reminders

1. HW 3.2 due Friday 02/25 11:59 pm
2. Mid-semester write up due Tuesday 03/15 11:59 pm
3. Exam #2 on March 03/29

   Sections on Exam 2: 3.1, 3.2, 3.3, 3.4, 10.1, 10.2, 10.3, 10.5

Remark

A logical statement having n component statements will have 2^n rows in its truth table

Exercise

Show $\sim p \vee \sim q$ is equivalent to $\sim(p \wedge q)$

p	q	$\sim p$	$\sim q$	$\sim p \vee \sim q$	$(p \wedge q)$	$\sim (p \wedge q)$
T	T	F	F	F	T	F
T	F	F	T	T	F	T
F	T	T	F	T	F	T
F	F	T	T	T	F	T

$\sim p \vee \sim q \equiv \sim (p \wedge q)$

Equivalent Statements

Two statements are equivalent if they have the same truth values

Exercise

show $\sim p \wedge \sim q \equiv \sim(p \vee q)$

p	q	$\sim p$	$\sim q$	$\sim p \wedge \sim q$	$p \vee q$	$\sim(p \vee q)$
T	T	F	F	F	T	F
T	F	F	T	F	T	F
F	T	T	F	F	T	F
F	F	T	T	T	F	T

$\sim p \wedge \sim q \equiv \sim(p \vee q)$

Exercise

Find a negation of each statement by using De Morgan's law

$$\sim(p \vee q) \equiv \sim p \wedge \sim q$$

$$\sim(p \wedge q) \equiv \sim p \vee \sim q$$

① I laughed or I cried

$(p \vee q)$

p = I laughed

q = I cried

$\sim p$ = I did not laugh

$\sim q$ = I did not cry

$\sim(p \vee q) \equiv \sim p \wedge \sim q$

I did not laugh and I did not cry

② You can pay me now or you can pay me later