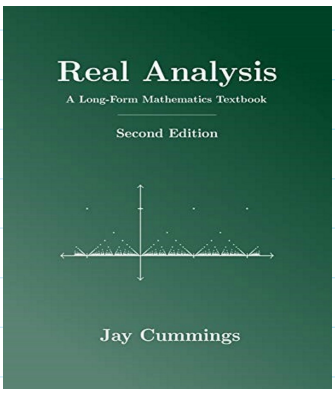
Thurs_02_03

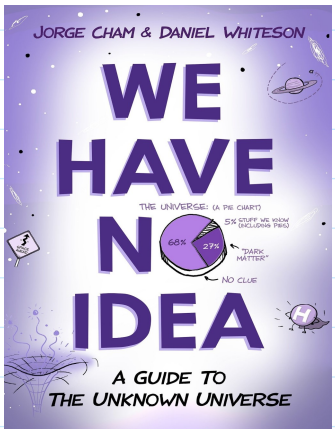
Thursday, February 3, 2022 1:13 PM

Set Operations



Here is a quote from the book 'Real Analysis' by Jay Cummings

Example #628 of why English is a confusing language: The word 'and' can mean either a union or an intersection. 'Students and faculty are welcome to attend' means the union of the students and the faculty are invited. But, 'If you are taking algebra and analysis this year, you'll enjoy this lecture' refers the intersection of those taking and those taking analysis. Worse still, 'and' can refer to quantities as well, not just to collections! 'These three pizzas and those five make eight.' Mathematicians use symbols to avoid these ambiguities.
The word 'or' is better.



Suppose you go to your favorite dinner and the waiter says 'Tea or Coffee'

For us in mathematics 'or' means you get both

Reminders

1. HW 2.1, 2.2 due Fri 02/04, 11:59 pm
2. EX #1 on 02/15
3. Study guide available on classnote page (check D2L)

Set Operations

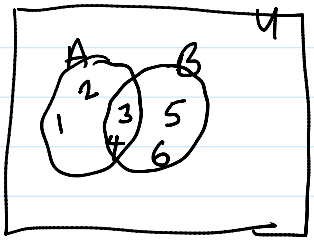
Intersection of Sets

Suppose A, B are sets, the intersection of A and B is the set of elements common to both A and B

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

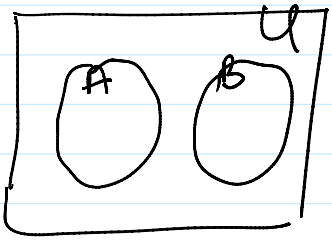
Example

1. $A = \{1, 2, 3, 4\}$ $B = \{3, 4, 5, 6\}$



$$A \cap B = \{3, 4\}$$

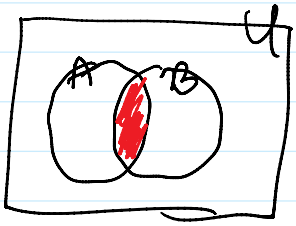
2.



Here Sets A, B are disjoint

So $A \cap B = \phi$

3.



Here the shaded portion is

$A \cap B$

Union of Sets

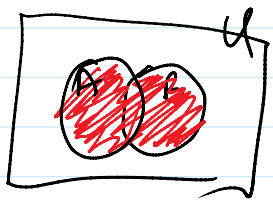
Let A, B be sets, the union of A, B written $A \cup B$ is the set of all elements belonging to either A or B

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

Example

1. $A = \{2, 5, 6\}$, $B = \{4, 6, 8, 7\}$

$A \cup B = \{2, 5, 6, 4, 8, 7\}$

2. 

The shaded portion is $A \cup B$

Example Find intersection and union of Complements

Complement of a Set

Suppose A is a set belonging to a universal set U then the complement of A $A' = \{x \mid x \in U \text{ and } x \notin A\}$

Exercise

$U = \{1, 2, 3, 4, 5, 6, 9, 10\}$

$A = \{1, 2, 5\}$ $B = \{2, 5, 7\}$ $C = \{1, 5, 6, 8\}$

(a) $A' \cap B$ $A' = \{3, 4, 6, 9, 10\} \cap B = \phi$

(b) $B' \cup C'$ $B' = \{1, 3, 4, 6, 9, 10\} \cup C' = \{2, 3, 4, 9, 10\} = \{1, 3, 4, 6, 9, 10, 7\}$

(c) $A \cap (B \cup C')$ $A \cap (B \cup C' = \{2, 5, 7, 3, 4, 9, 10\}) = \{2, 5\}$

(d) $(B \cup C)'$ $B \cup C = \{2, 5, 7, 1, 6, 8\}$, $(B \cup C)' = \{3, 4, 9, 10\}$

Write out Set operation in words

$A \cap (B \cup C')$ Set of elements in A and elements in B or not in C.

Difference of Sets

Suppose A, B are sets, the set difference $A - B$ or $(A \setminus B)$

$$A - B = \{x \mid x \in A \text{ and } x \notin B\}$$

$$= A \cap B'$$

Exercise

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{2, 4, 5, 6, 7\}$

$B = \{4, 5, 6\}$

$C = \{3, 5, 8\}$

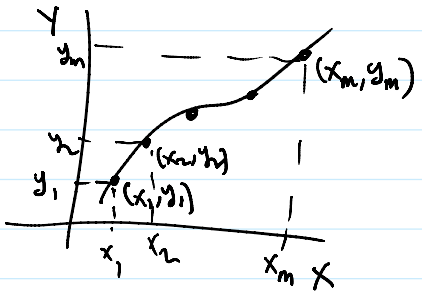
(a) $A - B = \{2, 7\}$

(b) $B - A = \phi = \{\}$

(c) $(A - B) \cup C' = (A-B) \cup C' = \{1, 2, 4, 6, 7, 9, 10\} = \{1, 2, 4, 6, 7, 9, 10\}$

Ordered pairs

Ordered pairs



Consider the graph of a curve in an x-y plane

Each point on this curve is an ordered pair

Let (a,b) be an ordered pair Then a is called the first component and b is called the second component

Cartesian product of a Set

Let A, B be sets the cartesian product of A and B

$$A \times B = \{(a,b) \mid a \in A \text{ and } b \in B\}$$

Example

$$A = \{1,2,3\} \quad , \quad B = \{4,5\}$$

$$A \times B = \{(1,4),(1,5),(2,4),(2,5),(3,4),(3,5)\}$$

$$B \times A = \{(4,1),(4,2),(4,3),(5,1),(5,2),(5,3)\}$$

Remark 1

$A \times B$ is equivalent to $B \times A$ since

$$n(A \times B) = n(B \times A)$$

Remark 2

$$n(A \times B) = n(A) \cdot n(B)$$

$$n(B \times A) = n(B) \cdot n(A)$$

Venn Diagrams

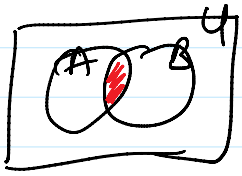
Let A, B be sets, let U be a universal set

1. Complement of A

$$A' = \{x \mid x \in U \text{ and } x \notin A\}$$

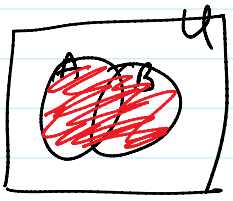
2. Intersection of A and B

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$



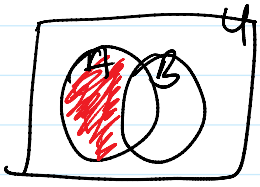
3. Union of A and B

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$



4. Set difference

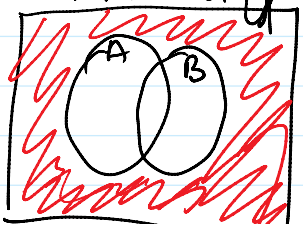
$$A - B = \{x \mid x \in A \text{ and } x \notin B\}$$



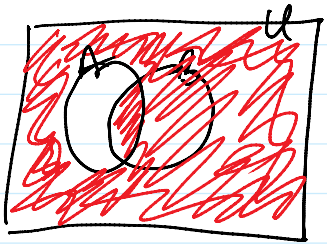
Exercise

Use Venn diagram to shade each set

1. $(A \cup B)'$



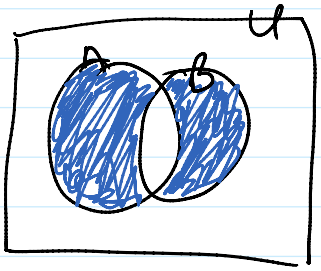
2.



$(A - B)'$

Solution to In-class problems

3.



$(A \cup B) - (A \cap B)$

4

$(A \cap B)' \cup C$