HW 1 Corrections

$\frac{1}{2} - \frac{1}{3}$

$\frac{3 \cdot 1}{3 \cdot 2} - \frac{1 \cdot 2}{3 \cdot 2}$

$\frac{3}{6} - \frac{2}{6}$

$\frac{3-2}{6} = \frac{1}{6}$

1. Find the domain of

$$f(x) = \frac{1}{x-2} - \frac{1}{x+3}$$

$$\frac{(x+3)}{(x+3)}\frac{1}{(x-2)} - \frac{1}{(x+3)}\cdot\frac{(x-2)}{(x-2)}$$

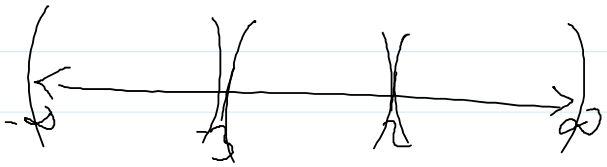
$$\frac{(x+3)-(x-2)}{(x+3)(x-2)}$$

$$\frac{x+3-x+2}{(x+3)(x-2)}$$

$$f(x) = \frac{5}{(x+3)(x-2)}$$

Domain: $(x+3)(x-2) \neq 0$

so $x \neq -3, 2$



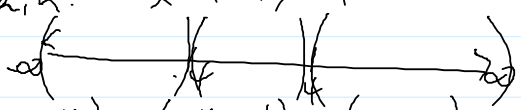
Domain: $(-\infty, -3) \cup (-3, 2) \cup (2, \infty)$

2. $f(x) = \frac{3x+2}{x^2-16}$, $g(x) = \frac{5x-4}{x^2-16}$

(i) $f+g(x) = f(x)+g(x) = \frac{3x+2}{x^2-16} + \frac{5x-4}{x^2-16} = \frac{3x+2+5x-4}{x^2-16}$

$= \frac{8x-2}{x^2-16}$

Domain: $x \neq 4, -4$



$(-\infty, -4) \cup (-4, 4) \cup (4, \infty)$

$3x+2 - \frac{5x-4}{\quad} = \frac{3x+2-(5x-4)}{\quad}$

(ii) $f-g(x) = f(x)-g(x) = \frac{3x+2}{x^2-16} - \frac{5x+4}{x^2-16}$ $\quad x^2-16$

$= \frac{3x+2-5x+4}{x^2-16} = \frac{-2x+6}{x^2-16}$

Domain: $x \neq 4, -4$

$(-\infty,-4) \cup (-4,4) \cup (4,\infty)$

(iii) $(fg)(x) = f(x) \cdot g(x)$

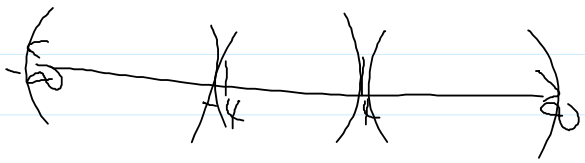
$= \frac{3x+2}{x^2-16} \cdot \frac{5x-4}{x^2-16}$

$= \frac{(3x+2)(5x-4)}{(x^2-16)(x^2-16)} = \frac{15x^2-12x+10x-8}{(x^2-16)^2}$

$= \frac{15x^2-2x-8}{(x^2-16)^2}$

Domain: $x \neq 4, -4$

$(-\infty,-4) \cup (-4,4) \cup (4,\infty)$



(iv) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\frac{3x+2}{x^2-16}}{\frac{5x-4}{x^2-16}}$

$= \frac{(3x+2)(x^2-16)}{(x^2-16)(5x-4)}$

Aside

$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \div \frac{c}{d}$

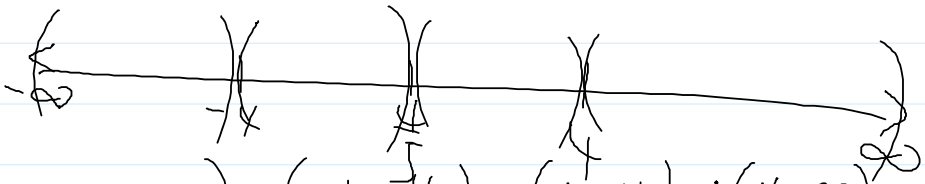
$= \frac{a}{b} \cdot \frac{d}{c}$

$= \frac{ad}{bc}$

Domain: $x \neq 4, -4, \frac{4}{5}$ $\quad 5x-4=0$

Domain: $x \neq 4, -4, \frac{4}{5}$

$5x - 4 = 0$
$5x = 4$
$x = \frac{4}{5}$



$(-\infty, -4) \cup (-4, \frac{4}{5}) \cup (\frac{4}{5}, 4) \cup (4, \infty)$

1.8 Inverse Functions

Example $f(\text{kayode}) = 2(\text{kayode}) + 3$

If $f(x) = 2x + 3$

$g(x) = \frac{1}{x}$

find $(f \circ g)(x) = f(g(x))$

$= f\left(\frac{1}{x}\right)$

$= 2\left(\frac{1}{x}\right) + 3$

$= \frac{2}{x} + 3$

Recall

If f, g are functions

$(f \circ g)(x) = f(g(x))$

$(g \circ f)(x) = g(f(x))$

$g(\text{kayode}) = \frac{1}{\text{kayode}}$

$f(x) = 2x + 3$

$g(x) = \frac{1}{x}$

find $(g \circ f)(x) = g(f(x))$

$= g(2x + 3)$

$= \frac{1}{2x + 3}$

In general,

$$(f \circ g)(x) \neq (g \circ f)(x)$$

When $(f \circ g)(x) = (g \circ f)(x)$

then g is the inverse function of f

and f is the inverse function of g

How do we find inverse function

If f is a function we denote the inverse of f with $\boxed{f^{-1}}$

Example

$f(x) = 2x+3$, $f^{-1}(x) = \dfrac{1}{2x+3}$ ✗

find the inverse of

$f(x) = 2x+3$

$f^{-1}(x) = \dfrac{x-3}{2}$

1. pick a number x
2. multiply by 2
3. add 3

1. pick a number x
2. subtract 3
3. divide by 2

Exercise

$f(x) = 2x+3$. $f^{-1}(x) = \dfrac{x-3}{2}$

$f(x) = 2x+3$, $f^{-1}(x) = \frac{x-3}{2}$

Show that

(i) $(f \circ f^{-1})(x) = x$

$$(f \circ f^{-1})(x) = f\left(f^{-1}(x)\right)$$
$$= f\left(\frac{x-3}{2}\right)$$
$$= 2\left(\frac{x-3}{2}\right)+3 = x-3+3 = x$$

(ii) $(f^{-1} \circ f)(x) = f^{-1}\left(f(x)\right)$

$$= f^{-1}(2x+3)$$
$$= \frac{2x+3-3}{2} = \frac{2x}{2} = x$$

How to find the inverse (textbook)

$f(x) = 2x+3$

Step 1: replace $f(x)$ with y

$y = 2x+3$

Step 2: Interchange x and y

$x = 2y+3$

Step 3: solve for y

$$x = 2y + 3$$
$$-3 \qquad -3$$
$$\frac{x-3}{2} = \frac{2y}{2}$$
$$\frac{x-3}{2} = y$$

step 4: replace y with f^{-1}

$$f^{-1}(x) = \frac{x-3}{2}$$

Find the inverse of

$$f(x) = \frac{2x-3}{x+1}$$

step 1: replace $f(x)$ with y

$$y = \frac{2x-3}{x+1}$$

step 2: interchange x and y

$$x = \frac{2y-3}{y+1}$$

step 3: Solve for y

$$x = \frac{2y-3}{y+1}$$
$$(y+1)x = \frac{2y-3}{(y+1)}(y+1)$$
$$(y+1)x = 2y-3$$

$$(y+1)x = 2y-3$$

$$yx + x = 2y - 3$$

$$-2y \qquad -2y$$

$$yx - 2y + x = -3$$

$$-x \qquad -x$$

$$yx - 2y = -3 - x$$

$$\frac{y(x-2)}{(x-2)} = \frac{-3-x}{(x-2)}$$

$$y = \frac{-3-x}{x-2}$$

$a(b+c) = a \cdot b + a \cdot c$

Step 4: replace y with $f^{-1}(x)$

$$f^{-1}(x) = \frac{-3-x}{x-2}$$

How to find inverse using graph

Claim 1

If point (a,b) is on the graph of f then point (b,a) is on the graph of f^{-1}

Claim 2

the graph of f and f^{-1} are symmetric about the diagonal $y=x$