Exam 2 MATH 1010 Spring 2022. Olumoyin

Name Answer Key

ESSAY. Write your answer in the space provided or on a separate sheet of paper.

Give the number of rows in the truth table for the compound statement.

1) $p \wedge (\sim q \vee r)$

There are 3 component statements

There are $2^3 = 8$ rows in the truth table

Construct a truth table for the statement.

2) $\sim(\sim(p \vee q))$

p	q	p∨q	~(p∨q)	~(~(p∨q))
T	T	T	F	T
T	F	T	F	T
F	T	T	F	T
F	F	F	T	F

$\sim(\sim(p \vee q)) = p \vee q$

Use De Morgan's laws to write the negation of the statement.

$\sim(p \wedge q) \equiv \sim p \vee \sim q$

3) A day late and a dollar short.

Not a day late or not a dollar short

Rewrite the statement using the <u>if...then</u> connective. Rearrange the wording or words as necessary.

4) All chocolate is good.

If it is chocolate, then it is good

5) A ship can't sail on land.

If this is a ship, then it can't sail on land.

Write the compound statement in words.
Let r = "The puppy is trained."
p = "The puppy behaves well."
q = "His owners are happy."

6) $\sim r \rightarrow \sim q$

If the puppy is not trained then his owners are not happy.

Write the compound statement in symbols.
Let r = "The food is good."
p = "I eat too much."
q = "I'll exercise."

7) If I exercise, then I won't eat too much.

$q \rightarrow \sim p$

Given p is true, q is true, and r is false, find the truth value of the statement.

8) $[(\sim p \rightarrow r) \wedge (\sim p \vee q)] \rightarrow r$

$[(\sim T \rightarrow F) \wedge (\sim T \vee T)] \rightarrow F$

$[(F \rightarrow F) \wedge (F \vee T)] \rightarrow F$

$[T \wedge T] \rightarrow F$

$T \rightarrow F$

F

Construct a truth table for the statement.

9) $(\sim p \rightarrow q) \rightarrow (q \rightarrow r)$

p	q	r	~p	(~p → q)	~r	(q → ~r)	(~p → q) → (q → ~r)
T	T	T	F	T	F	F	F
T	T	F	F	T	T	T	T
T	F	T	F	T	F	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	F	F	F
F	T	F	T	T	T	T	T
F	F	T	T	F	F	T	T
F	F	F	T	F	T	T	T

Write the negation of the conditional. Use the fact that the negation of $p \rightarrow q$ is $p \wedge \sim q$.

10) If you give your coat to the doorman, he will give you a dirty look.

You give your coat to the doorman and he will not give you a dirty look.

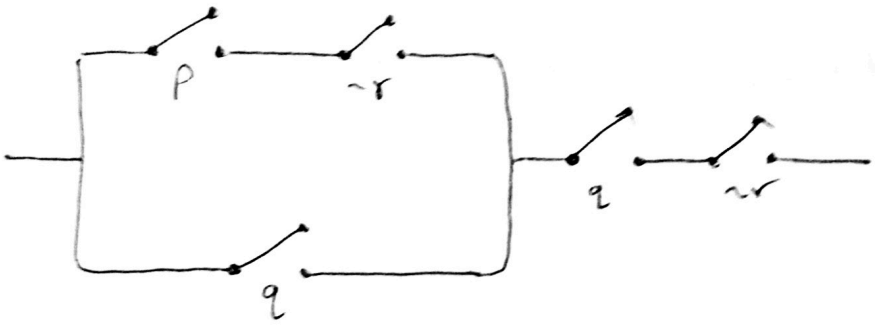
Write an equivalent statement that does not use the if ... then connective. Use the fact that $p \to q$ is equivalent to $\sim p \vee q$

11) If the sun comes out Tuesday, the roses will open.

The sun does not come out Tuesday or the roses will open

Draw a circuit representing the following statement as it is given. Simplify if possible.

12) $[(p \wedge \sim r) \vee q] \wedge (q \wedge \sim r)$



Write the converse, inverse, or contrapositive of the statement as requested.

13) If I pass, I'll party. $p \to q$ p: I pass, q: I'll party

Contrapositive $\sim q \to \sim p$

If I don't party, I didn't pass

14) All cats catch birds. $p \to q$ p: It is a cat, q: it catches birds

Inverse $\sim p \to \sim q$

If it is not a cat, it does not catch birds

15) If I were young, I would be happy. $p \to q$ p: I were young, q: I would be happy

Converse $q \to p$

If I were happy, I would be young

Rewrite the statement in the form "if p, then q".

16) I will lose weight if I diet.

If I diet, then I will lose weight

Given a group of students: G = {Allen, Brenda, Chad, Dorothy, Eric} or G = {A, B, C, D, E}, list and count the different ways of choosing the following officers or representatives for student congress. Assume that no one can hold more than one office.

17) Three representatives, if two must be male and one must be female

ACB, ACD, AEB, AED, CEB, CED

18) A president, a secretary, and a treasurer, if the president must be a woman and the other two must be men

BAC, BAE, BCA, BCE, BEA, BEC

DAC, DAE, DCA, DCE, DEA, DEC

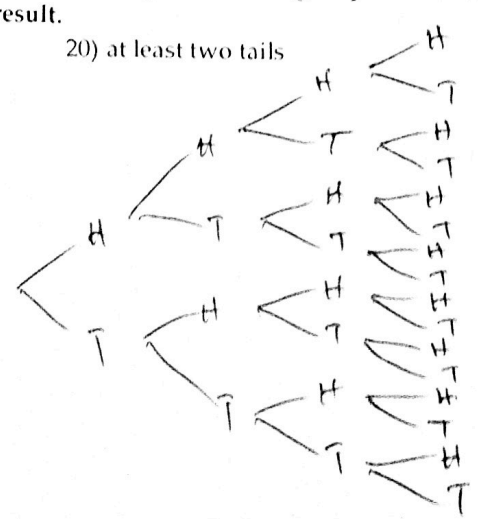
Solve the problem.

19) Construct a product table showing all possible two-digit numbers using digits from the set {2,3, 6, 8}.

first digit \ 2nd digit	2	3	6	8
2	22	23	26	28
3	32	33	36	38
6	62	63	66	68
8	82	83	86	88

Use a tree diagram showing all possible results when four fair coins are tossed. Then list the ways of getting the indicated result.

20) at least two tails



at least two tails = two tails, 3 tails, 4 tails)

two tails - HTTH, HHTT, TTHH, THHT, HTHT, THTH

3 tails - TTTH, TTHT, THTT, HTTT

4 tails - TTTT

Solve the problem.

21) At a lumber company, shelves are sold in 5 types of wood, 2 different widths and 6 different lengths. How many different types of shelves could be ordered?

FCP $\quad 5 \cdot 2 \cdot 6 = 60$ ways

22) A manager has 15 employees of the same ability. How many 10 employee groups can he create?

$15 C 10 = 3003$ groups

23) Three married couples have reserved six seats in a row at the theater, starting at an aisle seat. In how many ways can they arrange themselves if there are no restrictions on the seating arrangement?

Permutation exercise

$6 P 6 = 720$ arrangements

24) How many ways can a president, vice-president, secretary, and treasurer be chosen from a club with 12 members? Assume that no member can hold more than one office.

permutation exercise

$12 P 4 = 11880$ ways

25) The library is to be given 7 books as a gift. The books will be selected from a list of 25 titles. If each book selected must have a different title, how many possible selections are there?

Combination exercise

$25C7 = 480700$

26) A poker hand consists of 5 cards dealt from an ordinary deck of 52 playing cards. How many different hands are there consisting of three hearts and two spades?

$13C3 + 13C2 = 364$

27) If a single card is drawn from a standard 52-card deck, in how many ways could it be an ace or a spade?

Modify Quest #27 $n(A \text{ or } S) = n(A) + n(S) - n(A \text{ and } S)$
$= 4 + 13 - 1 = 16$

(28) If a single card is drawn from a standard 52-card deck, what is the probability it could be an ace or spade

The probability exercise was solved in class

Find the number of ways to get the following card combinations from a 52-card deck.

28) All face-cards in a five-card hand

There are 12 face cards in a 52-card deck

$12C5 = 792$

Solve the problem.

29) If a license plate consists of six digits, how many different licenses could be created having at least one digit repeated. (Bonus)

A = at least one digit repeated

Modify Quest #29 $n(A) = n(S) - n(A')$
$= 10^6 - (10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5) = 848800$

(29) If a license plate consists of six digits, what is the probability of a license plate having at least one digit repeated.

The probability exercise was solved in class