Thursday, November 5, 2020 8:22 AM

4.5 Curve Sketching (continuation of 4.3)

(we say how f' helps us sketch f)

(4.3 + some college + x Algebra ↑ new stuff)

Checklist when you want to sketch a function

① Domain of the function

example polynomial functions - Domain is all real numbers $(-\infty, \infty)$

$f(x) = e^x$ $(-\infty, \infty)$

$f(x) = \sqrt{x}$ $(0, \infty)$

$f(x) = \frac{1}{x}$ $(-\infty, 0) \cup (0, \infty)$

$f(x) = \ln(x)$ $(0, \infty)$

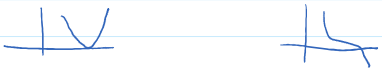
$f(x) = \frac{1}{x-2}$ $(-\infty, 2) \cup (2, \infty)$

② Intercept (we find y-intercept by computing $f(0)$ / we find x-intercept by setting $f(x) = 0$, solving for x)

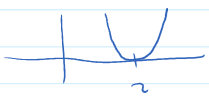
(x-intercepts or y-intercepts)

⇓

is where the curve either touches or crosses the x-axis



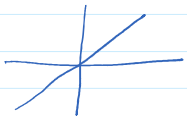
$f(x) = x^2$  x-intercept $x = 0$ (multiplicity of 2)

$f(x) = (x-2)^2$  x-intercept $x = 2$ (multiplicity of 2)

(x-intercept is going to touch the x-axis if the multiplicity is even)

⇓

(The number of times the x-intercept occurs)

$f(x) = x$  x-intercept $x = 0$ (multiplicity of 1)

$f(x) = x-2$  x-intercept $x = 2$ (multiplicity of 1)

$f(x) = x^3$  x-intercept $x = 0$ (multiplicity of 3)

(c) Symmetry

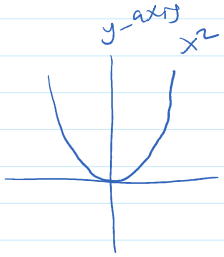
(a) Symmetric about the y-axis (reflexional symmetry) (Even functions)

$$f(-x) = f(x)$$

example

$$f(x) = x^2$$

$$f(-x) = (-x)^2 = (-x)\cdot(-x) = x^2$$



(b) Symmetric about the origin (rotational symmetry) (odd functions)

$$f(-x) = -f(x)$$

Example

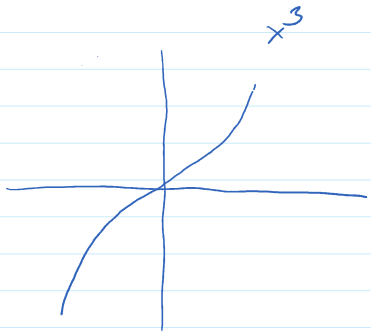
$$f(x) = x^3$$

$$f(-x) = (-x)^3 = (-x)\cdot(-x)\cdot(-x)$$

$$= x^2 \cdot (-x)$$

$$= -x^3$$

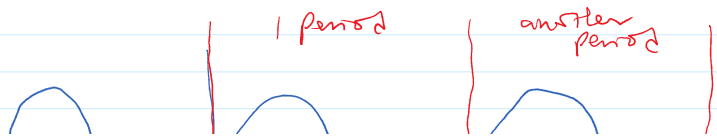
$$= -f(x)$$



(d) Periodic function

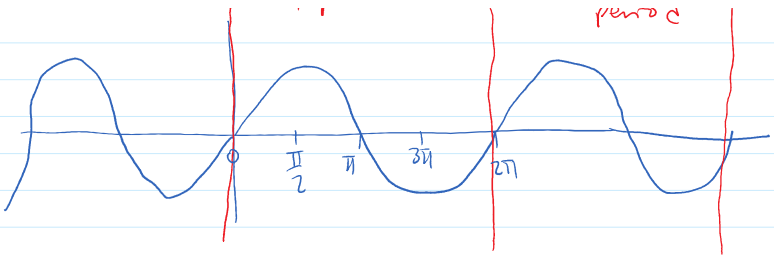
$f(x+p) = f(x)$ for every x in Domain of f

example

$\sin(x) = \sin(x + 2\pi)$ 

example

$\sin(x) = \sin(x + 2\pi)$



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(7) Asymptotes

Vertical Asymptotes

$\lim_{x \to \pm a} f(x) = \pm\infty$ then line $x = \pm a$ is a Vertical asymptote

Horizontal Asymptotes

$\lim_{x \to \infty} f(x) = L$ then line $y = L$ is an Horizontal asymptote

Slant / oblique Asymptote

Example

find the Asymptote of

$$f(x) = \frac{x^3}{x^2+1}$$

$3 > 2$ (deg of numerator > deg of denominator)

no H.A

we have a slant asymptote

$$x^2+1 \overline{)\, x^3}$$

$$-(x^3 + x)$$

$$-x$$

$$\frac{x^3}{x^2+1} = x - \frac{x}{x^2+1}$$

Slant Asymptote

Casual definition of horizontal asymptote

$$f(x) = \frac{P(x)}{Q(x)}$$

$P(x) = \boxed{a x^n} + \cdots$

$Q(x) = \boxed{b x^m} + \cdots$

If $n < m$

then line $y = \frac{a}{b}$ is the horizontal asymptote

If $n > m$

there is no Horizontal asymptote

If $\lim_{x \to \infty} \left(f(x) - x \right) = 0$ then $y = x$ is the slant asymptote