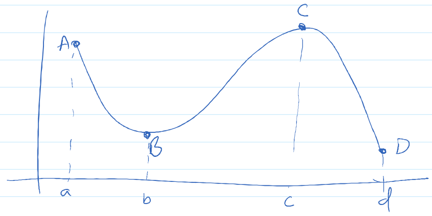
4.3 What does $\boxed{f'}$ tell us about $\boxed{f}$

f': derivative of a function

f: help us to sketch f



f

Q Can f' find portions of the domain of f where f is increasing or decreasing

(i) $f'(x) > 0$ (slope) in B to C, f is increasing on (b, c)

(ii) $f'(x) < 0$ in A to B, f is decreasing on (a, b)

(iii) $f'(x) < 0$ in C to D, f decreasing on (c, d)

Increasing / Decreasing Test

(i) If $f'(x) > 0$ on an interval, f is increasing on that interval

(ii) If $f'(x) < 0$ on an interval, f is decreasing on that interval

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Example

$$f(x) = 3x^4 - 4x^3 - 12x^2 + 5$$

Find the portion of the domain of f where f is increasing or decreasing

Solution

$$f'(x) = 12x^3 - 12x^2 - 24x$$
$$= 12x(x^2 - x - 2)$$
$$f'(x) = 12x(x-2)(x+1)$$

To partition the domain, we need to find the critical points

Set $f'(x) = 0$, solve for x

$12x(x-2)(x+1) = 0$, solve for x

$12x = 0$ or $x - 2 = 0$ or $x + 1 = 0$

$x = 0$ or $x = 2$ or $x = -1$

So the critical points are

$-1, 0, 2$

aside

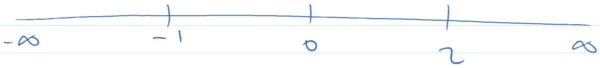
Zeroth product property

If $a.b = 0$ then either $a = 0$, $b = 0$, or both are zero

a, b are real.

so the critical points are

$-1, 0, 2$



or both are zero

a,b are real numbers

If a,b were not real numbers say a,b are from $\mathbb{Z}_6 = \{0,1,2,3,4,5\}$

$2 \cdot 3 \equiv 0 \pmod 6$

Next, we use the increasing/decreasing test

$f'(x) = 12x(x-2)(x+1)$

x	$12x$	$x-2$	$x+1$	$f'(x)$	f
$x < -1$	$-ve$	$-ve$	$-ve$	$-ve$	decreasing on $(-\infty,-1)$
$-1<x<0$	$-ve$	$-ve$	$+ve$	$+ve$	increasing on $(-1,0)$
$0<x<2$	$+ve$	$-ve$	$+ve$	$-ve$	decreasing on $(0,2)$
$x>2$	$+ve$	$+ve$	$+ve$	$+ve$	increasing on $(2,\infty)$

negative = $-ve$

positive = $+ve$

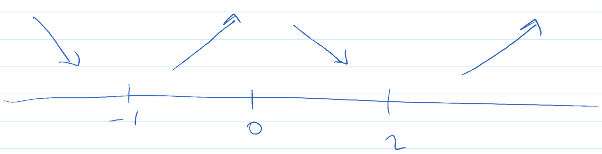
Algebra

$-ve \cdot -ve = +ve$

$+ve \cdot +ve = +ve$

$-ve \cdot +ve = -ve$

$+ve \cdot -ve = -ve$



f has a local max at 0

f has a local min at -1

f has a local min at 2

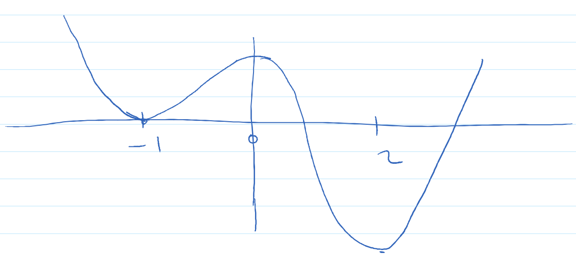
First derivative Test

Suppose c is a critical point

(a) If f' changes from positive to negative at c, f has a local maximum at c

(b) If f' changes from negative to positive at c, f has a local minimum at c

$f(x) = 3x^4 - 4x^3 - 12x^2 + 5$



local min at $x=-1$

$f(-1) = 0$

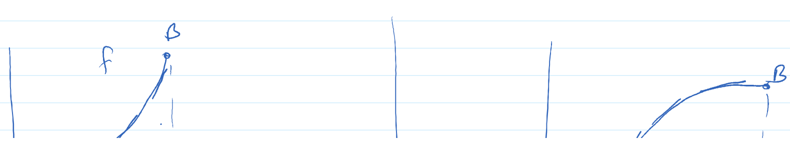
local max at $x=0$

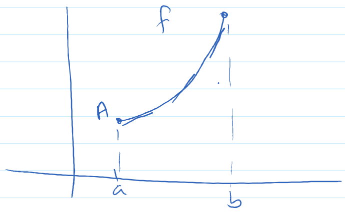
$f(0) = 5$

local min at $x=2$

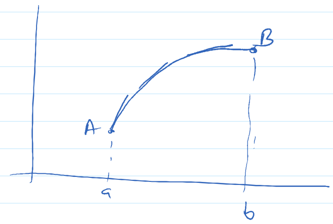
$f(2) = -27$

Concavity





If a curve lies above tangent
f is called concave upward on (a,b)



If a curve lies below tangent
f is called concave downward on (a,b)

Concavity Test

(a) If $f''(x) > 0$ on an interval, then f concave upward on that interval

(b) If $f''(x) < 0$ on an interval, then f concave downward on that interval

Definition

A point P on a curve f is called an inflection point if f is continuous at P and the curve

(i) changes from concave upward to concave downward at P
(ii) changes from concave downward to concave upward at P