4.2 The Mean Value Theorem

Rolle's Theorem:

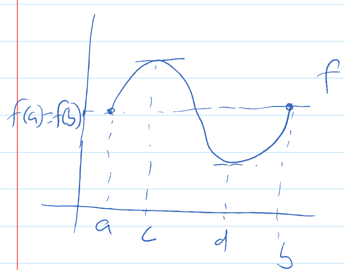
Let (i) f be continuous on $[a,b]$

(ii) f differentiable on (a,b)

(iii) $f(a)=f(b)$

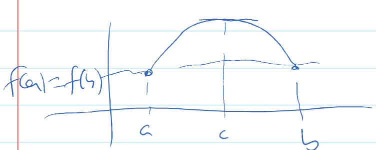
Then there exist a c in (a,b) such that

$$f'(c)=0$$

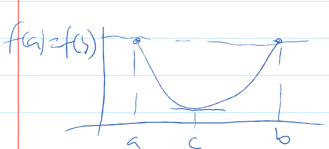


$f'(c)=0$ (There is an horizontal line to the f at c)

$f'(d)=0$



$f'(c)=0$



$f'(c)=0$

Mean Value Theorem

Let

(i) f be continuous on $[a,b]$

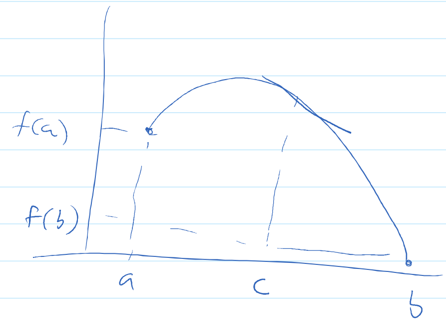
(ii) f be differentiable on (a,b)

Then there exist a c in (a,b) such that

$$f(b)-f(a)=f'(c)(b-a)$$

or

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$



$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

Example on the mean value Theorem

consider $f(x) = x^3 - x$

verify with your calculator

$f(x) = x^3 - x$ is continuous on $[0, 2]$

$f(x) = x^3 - x$ is differentiable on $(0, 2)$ (since it is a polynomial)

So by mean value Theorem, there must exist a c in $(0, 2)$ such that

$$f(2) - f(0) = f'(c)\,(2-0) \quad \text{———} \quad (*)$$

$f(0) = 0^3 - 0 = 0$

$f(2) = 2^3 - 2 = 6$

$f'(x) = \frac{d}{dx}(f(x)) = \frac{d}{dx}(x^3 - x) = 3x^2 - 1$

$f'(c) = 3c^2 - 1$

So $(*)$ becomes

$$6 - 0 = 3c^2 - 1 \; (2-0)$$

$$6 = 6c^2 - 2$$

$$0 = 6c^2 - 8$$

$$0 = 2(3c^2 - 4)$$

$$3c^2 - 4 = 0$$

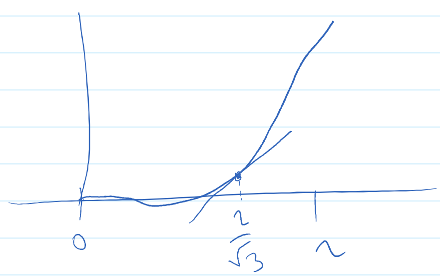
$$3c^2 = 4$$

$$c^2 = \frac{4}{3}$$

$$c = \pm\sqrt{\frac{4}{3}} = \pm\frac{2}{\sqrt{3}}$$

c must be $\frac{2}{\sqrt{3}}$

So if you graph $x^3 - x$ on $[0, 2]$



$$f(2) - f(0) = f'(c) \cdot (2 - 0)$$

here we found

$c = \frac{2}{\sqrt{3}}$

Theorem (A)

If $f'(x) = 0$ for all x in (a,b) then f is a constant on (a,b)

Aside

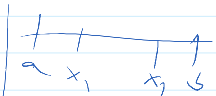
$f(x) = 5$, $\frac{d}{dx}(f(x)) = 0$

Pf.

Choose x_1, x_2 in (a,b) such that $x_1 < x_2$

given f is differentiable on (a,b) so

- f is also differentiable on (x_1, x_2)
- f is continuous on $[x_1, x_2]$



So by the mean value Theorem, there exist a c in (x_1, x_2) such that

$$f(x_2) - f(x_1) = f'(c) \cdot (x_2 - x_1)$$

Since $f'(x) = 0$ for every x in (a,b), so $f'(c) = 0$

$$f(x_2) - f(x_1) = 0$$

$$f(x_2) = f(x_1)$$

So f must be a constant

Exam #3 practice set

$\boxed{3.4}$, 3.5, 3.6, $\boxed{3.7, 3.8, 3.9}$, 4.1

$\boxed{\#26}$ Find the absolute maximum and absolute minimum of f

$$f(x) = 16 + 4x - x^2 \quad \text{on } [0,5]$$

use the closed interval method

(i) find the critical value of f at critical value of f in $(0,5)$

(ii) find the values of f at the endpoints

max of (i), (ii) is absolute maximum

min of (i), (ii) is absolute minimum

(i) To find critical value

$$f'(x) = 4 - 2x = 0$$

$$\Rightarrow x = 2 \quad \text{critical value}$$

$$f(2) = 16 + 4(2) - 2^2 = 20$$

(ii) $f(0) = 16 + 4(0) - 0^2 = 16$

$f(5) = 16 + 4(5) - 5^2 = 16 + 20 - 25 = 11$

max of (i), (ii) is $\underline{\underline{20}}$. $f(2) = 20$

min of (i), (ii) is $\underline{\underline{11}}$, $f(5) = 11$