Wednesday, October 21, 2020 7:41 AM

Sections due on WebAssign this weekend 10/24

3.7, 3.8, 3.9

Chapter 4 Application of Differentiation

* Optimization (theme)

'Best' way to use up resources

optimize cost (minimize)

optimize time

profit (maximize)

How much will it cost

Suppose there is a vaccine today

* who do you give first

*

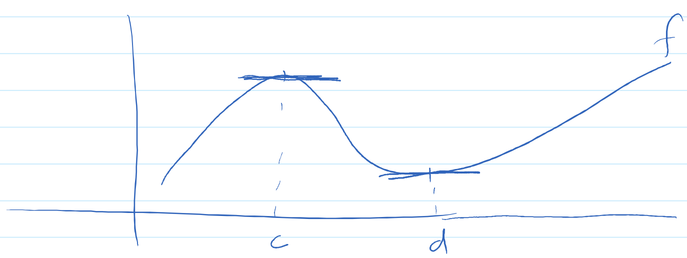
*

Machine Learning

gradient descent (Optimization, ...)

4.1 Maximum and minimum values

If given a function f(x)



A lesson from Calculus so far

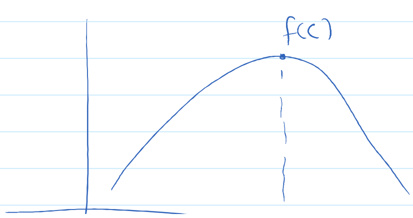
If you can draw an horizontal line to f at a point c

then $f'(c) = 0$

Definition Let c be a number in the Domain of f. Then f(c)

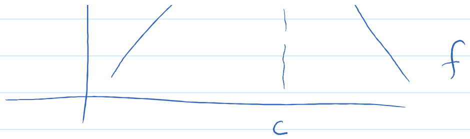
1. Global maximum (Absolute maximum).

of f on its domain if $f(c) \geq f(x)$ for x in Domain of f

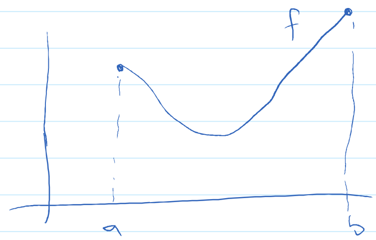


$f(c) > f(x)$ for all x in domain of f

So f(c) an absolute maximum at c
(global)



so $f(c)$ a absolute maximum at c (global)

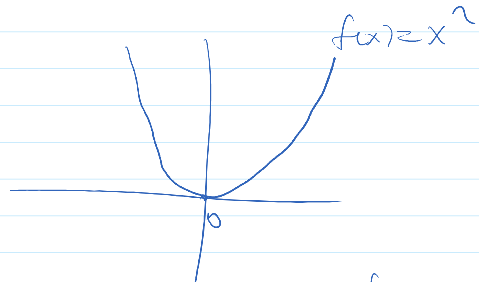


Domain of f is $[a,b]$

$f(b)$ is an absolute maximum or global maximum at b

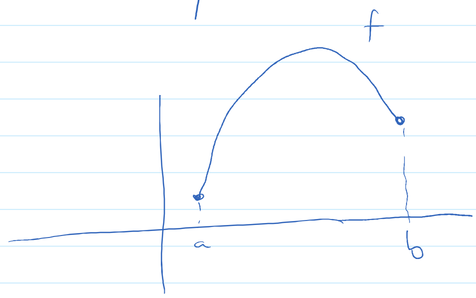
2. Global minimum (Absolute Minimum)

if $f(c) \le f(x)$ for all values of x in Domain of f



$f(0) \le f(x)$ for all x in the Domain of f

$f(0)$ is the absolute (global) minimum at 0



$f(a) \le f(x)$ for all x in domain of f

$f(a)$ is the absolute (global) Minimum at a

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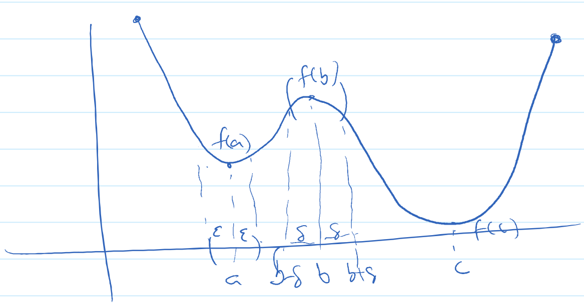
local minimum and local maximum

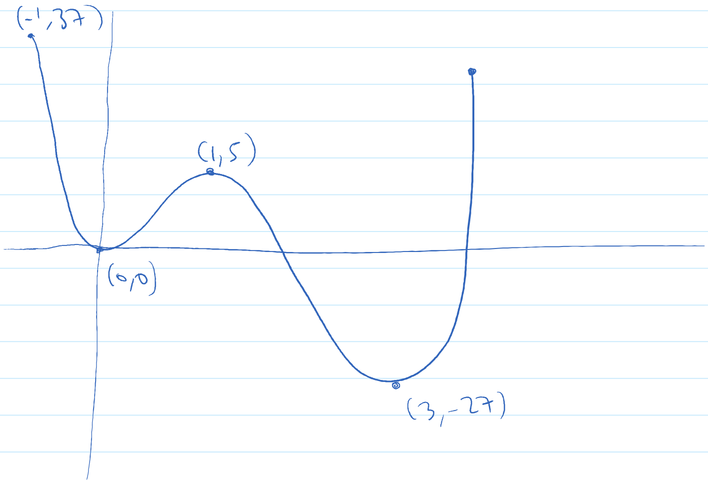


$f(b)$ is a local maximum in $(b-\delta, b+\delta)$, $\delta > 0$

$f(a)$ is a local minimum in $(a-\varepsilon, a+\varepsilon)$, $\varepsilon > 0$

$f(a)$ is a local minimum if $f(a) \le f(x)$ for x values near a

$f(b)$ is a local maximum if $f(b) \geq f(x)$ for x values near b



local minimum

$f(0) = 0$ minimum for points near 0

local maximum

$f(1) = 5$ maximum for points near 1

global maximum

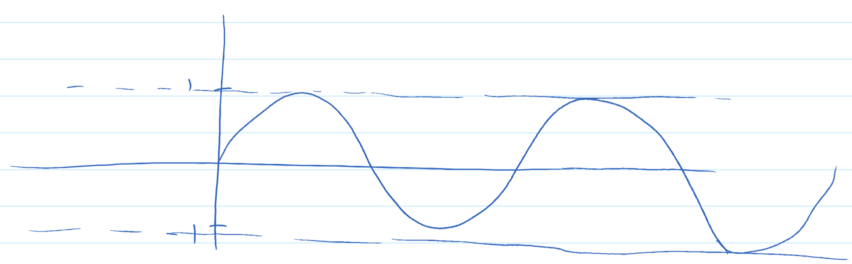
$f(-1) = 37$

global minimum

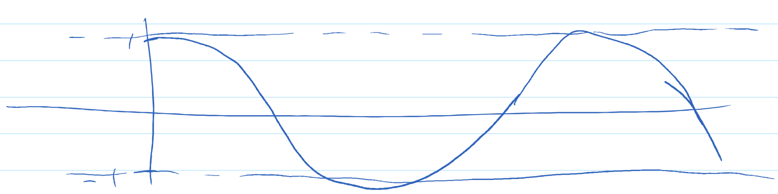
$f(3) = -27$

Extreme values (global maximum, global minimum)

Recall, $f(x) = \sin(x)$



$f(x) = \cos(x)$



1 is a local and global maximum

-1 is a local and global minimum

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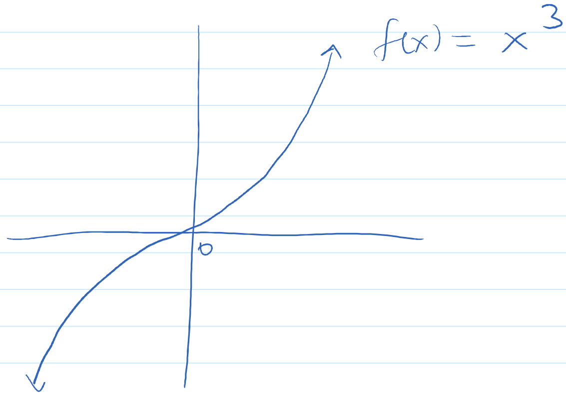
let us see a function that does not have a local or global maximum / minimum



$f(0) \not\leq f(x)$ for x in domain of f

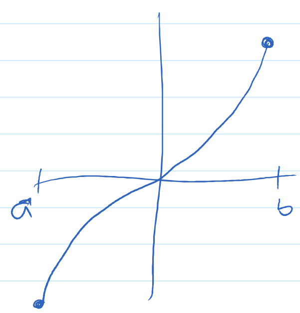
$f(0) \not\geq f(x)$ for x in domain of f

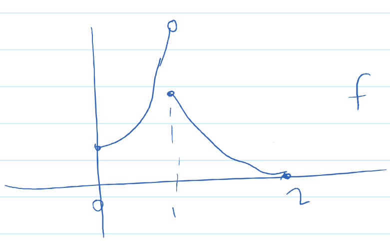
<u>Theorem</u> (Extreme Value Theorem)

If f is <u>continuous</u> on a <u>closed interval</u> $[a,b]$

then f attains an absolute maximum value $f(c)$ and an absolute minimum $f(d)$ for some c, d in $[a,b]$

$f(x) = x^3$ on



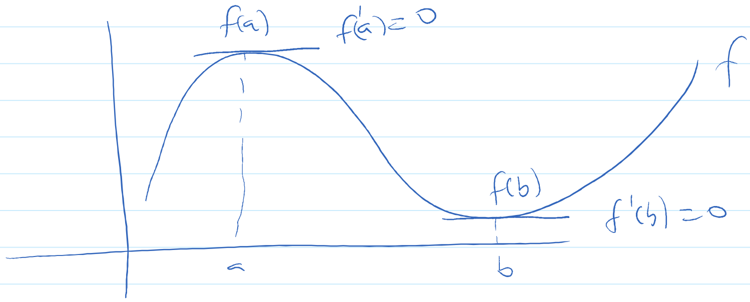


<u>Fermat's Theorem</u>

If f has a local maximum or minimum at c, and if $f'(c)$ exists

$\Downarrow$

then $f'(c) = 0$



$f(a)$ is a local maximum

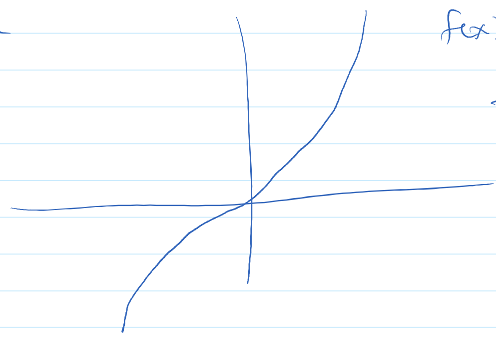
$f(b)$ is a local minimum

Caution with using <u>fermat's theorem</u>

Even if $f'(c) = 0$ for some c in the domain of f there may not be a local maximum or minimum at c

The <u>converse</u> of fermat's Theorem is not true in general

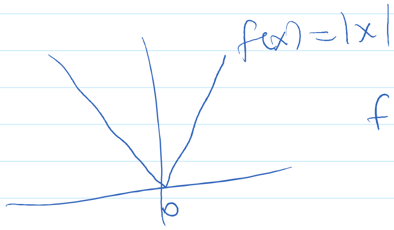
Example



$f(x) = x^3$

$f'(x) = 3x^2$

$f'(0) = 0$ but we established that $f(0)$ is not a local maximum or local minimum for $f = x^3$



$f'(0)$ does <u>not exist</u>

<u>Aside</u>.

If $f'(c) = 0$

Then $\boxed{c}$, it is possible $f(c)$ is a local max or min

it is also possible that it is not

Definition

A critical value/number of a function is a number c in the domain of f such that either $f'(c) = 0$ or $f'(c)$ does not exist

I encourage that you look up this materral in your text book

Corollary to Fermat's Theorem

If f has a local maximum or minimum at c, then c is a critical number of f

The closed Interval method

(help us find absolute (global) maximum and minimum) of values of a continuous function f on a closed interval $[a, b]$

1. find the value of f at the critical numbers of f in (a, b)
2. find the values of f at a, b
3. The largest value from step 1 and 2 is the absolute maximum and the smallest value from step 1 and 2 is the absolute min

Example

Find the absolute maximum and minimum of the function

$$f(x) = x^3 - 3x^2 + 1 \qquad -\frac{1}{2} \le x \le 4$$

we know f is continuous on $[-\frac{1}{2}, 4]$

first, find the critical numbers of f

Critical numbers of f are c in $[-\frac{1}{2}, 4]$ such that

$$f'(c) = 0 \quad \text{or} \quad f'(c) \text{ DNE}$$

$$f'(x) = 3x^2 - 6x = 0$$

$$3x^2 - 6x = 0$$

$$3x(x-2) = 0$$

$$f'(0) = 0$$

$$f'(2) = 0$$

$x = 0$ or $x = 2$

So $0, 2$ are critical values

step 1

$f(x) = x^3 - 3x^2 + 1$

$f(0) = 0^3 - 3(0)^2 + 1 = 1$

$f(2) = 2^3 - 3(2)^2 + 1 = 8 - 12 + 1 = -3$

step 2

$f(x) = x^3 - 3x^2 + 1$, the end points are $-\frac{1}{2}, 4$

$f(-\frac{1}{2}) = \left(-\frac{1}{2}\right)^3 - 3\left(-\frac{1}{2}\right)^2 + 1 \qquad = -\frac{1}{8} - 3\left(\frac{1}{4}\right) + 1$

$= \frac{1}{8} \qquad\qquad = -\frac{1}{8} - \frac{3}{4}\cdot\frac{2}{2} + \frac{1 \cdot 8}{8}$

$= -\frac{1}{8} - \frac{6}{8} + \frac{8}{8} = \frac{-1 - 6 + 8}{8} = \frac{1}{8}$

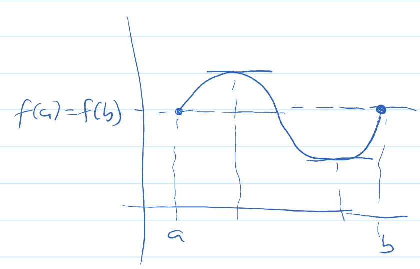
$f(4) = 4^3 - 3(4)^2 + 1 = 17$

absolute maximum is $f(4) = 17$ (max of step 1 & 2)

absolute minimum is $f(2) = -3$ (min of step 1 & 2)

4.2 The Mean Value Theorem

Rolle's Theorem



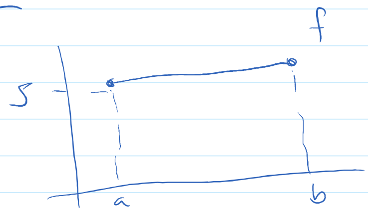
Let f satisfy the following assumptions

1. f is continuous on $[a, b]$
2. f is differentiable on (a, b)
3. $f(a) = f(b)$

Then there is a number c in (a, b) such that $f'(c) = 0$

Then there is a number c in (a,b) such that $f'(c)=0$

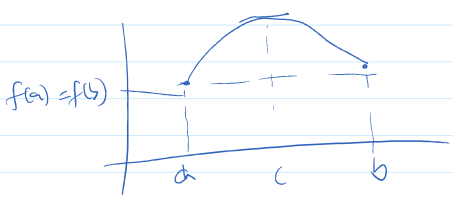
Example



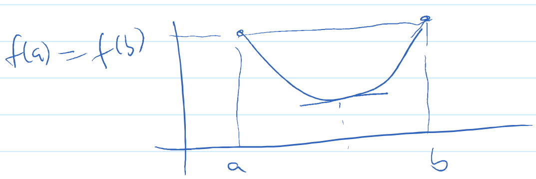
$f(x)=5$ on $[a,b]$

$f'(x)=0$

$f'(c)=0$ for all c in (a,b)



$f'(c)=0$ on (a,b)

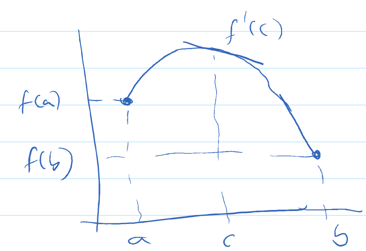


$f'(c)=0$ on (a,b)

The mean Value Theorem

Let f be function that satisfies the following assumptions

1. f is continuous on $[a,b]$
2. f is differentiable on (a,b)



So, there is a number c in (a,b)

such that $$f'(c)=\frac{f(b)-f(a)}{b-a}$$

$$f(b)-f(a)=f'(c)\,(b-a)$$

(The rolle's thm is a particular case of the mean Value Theorem).

We have Exam #3 next Wednesday

Exam #3 practice set

Sections on Exam #3

3.4, 3.5, 3.6, 3.7, 3.8, 3.9, 4.1

HW due 10/29

3.7, 3.8, 3.9