Sections due on WebAssign this weekend 10/24

3.7, 3.8, 3.9

Chapter 4 Application of Differentiation

* Optimization (theme)

'Best' way to use up resources

optimize cost —

optimize time

How much will it cost

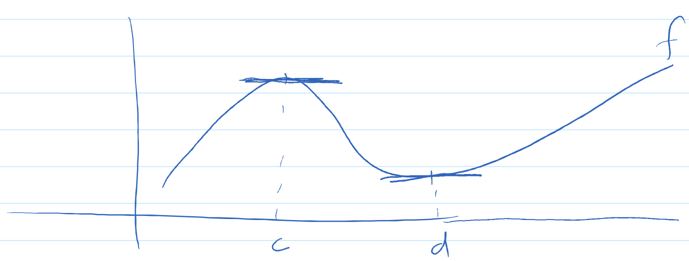
Suppose there is a vaccine today
* who do you give first
*
*

Machine Learning

gradient descent (Optimization, ...)

4.1 Maximum and minimum values

If given a function $f(x)$

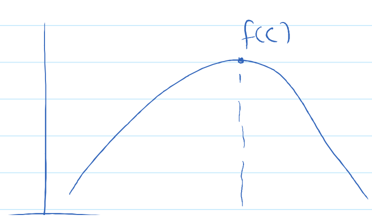


A lesson from Calculus so far

If you can draw an horizontal line to f at a point c then $f'(c) = 0$

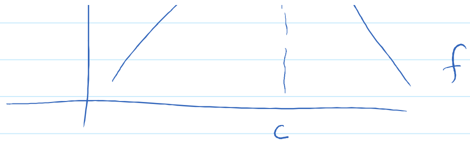
Definition Let c be a number in the Domain of f. Then $f(c)$

1. Global maximum (Absolute maximum). of f on its domain if $f(c) \geq f(x)$ for x in Domain of f

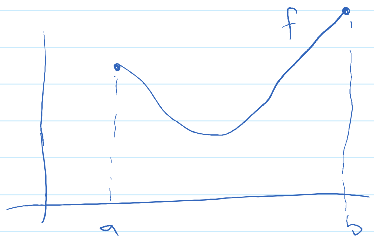


$f(c) > f(x)$ for all x in domain of f

so $f(c)$ an absolute maximum



so $f(c)$ a absolute maximum

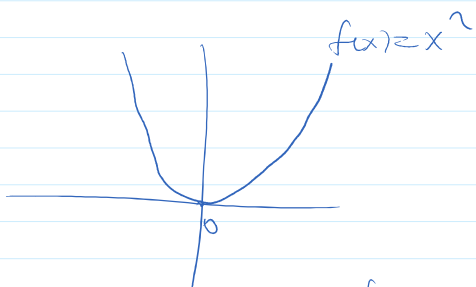


Domain of f is $[a, b]$

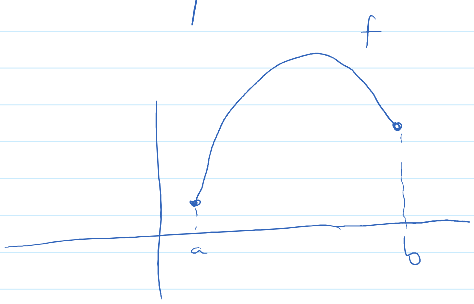
$f(b)$ is an absolute maximum or global maximum

2. Global minimum (Absolute minimum)

If $f(c) \leq f(x)$ for all values of x in Domain of f



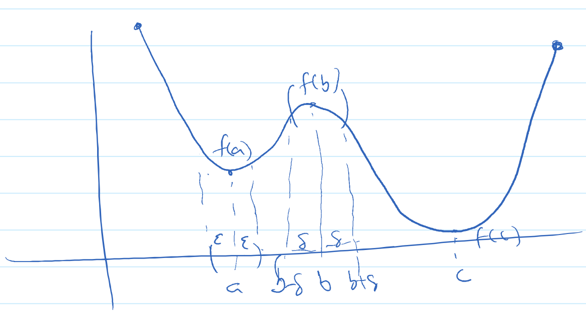
$f(0) \leq f(x)$ for all x in the Domain of f



$f(a) \leq f(x)$ for all x in domain of f

---

local minimum and local maximum



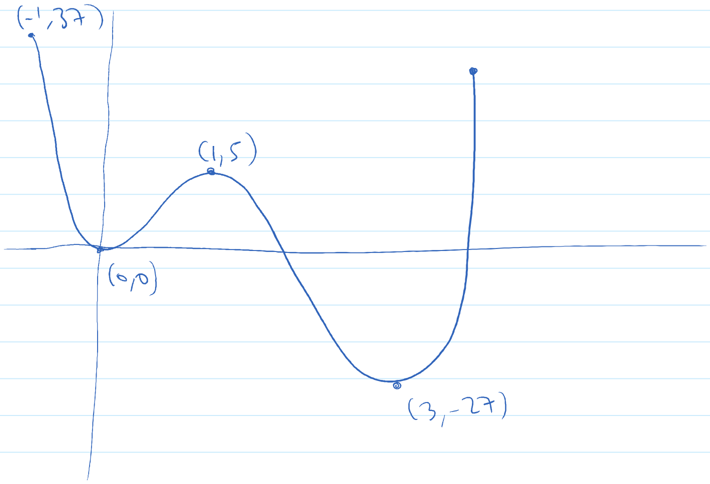
$f(b)$ is a local maximum in $(b-\delta, b+\delta)$, $\delta > 0$

$f(a)$ is a local minimum in $(a-\varepsilon, a+\varepsilon)$, $\varepsilon > 0$

$f(a)$ is a local minimum if $f(a) \leq f(x)$ for x values near a

$f(b)$ is a local maximum if $f(b) \geq f(x)$ for x values near b

---



local minimum

$f(0) = 0$ minimum for points near 0

local maximum

$f(1) = 5$ maximum for points near 1

global maximum

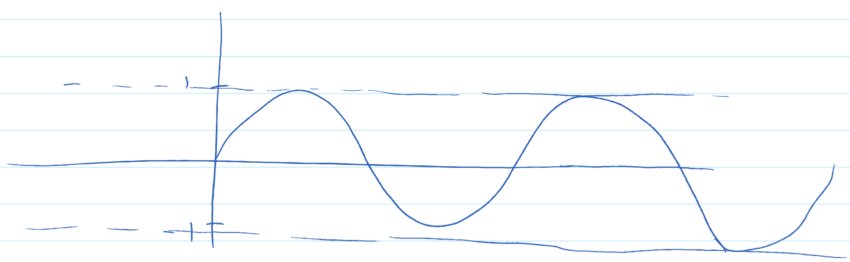
$f(-1) = 37$

global minimum

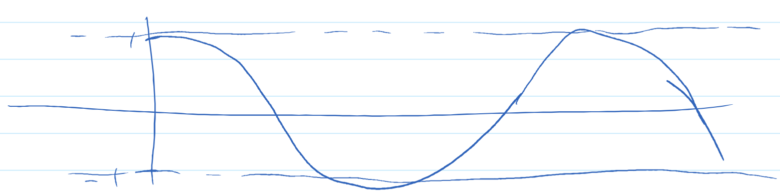
$f(3) = -27$

Extreme values (global maximum, global minimum)

Recall, $f(x) = \sin(x)$



$f(x) = \cos(x)$

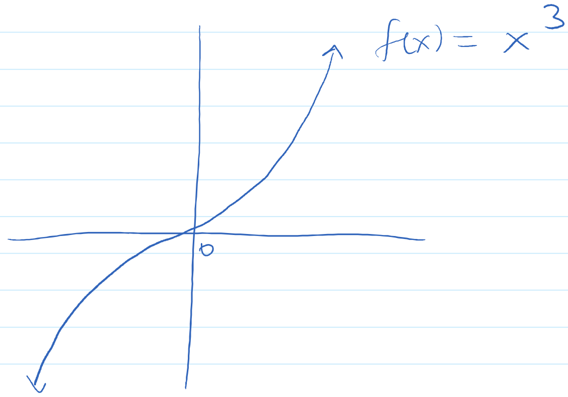


1 is a local and global maximum

-1 is a local and global minimum

---

let us see a function that does not have a local or global maximum/minimum



$f(0) \not\leq f(x)$ for x in domain of f

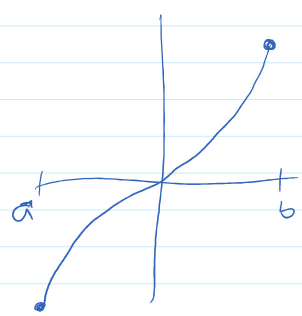
$f(0) \not\geq f(x)$ for x in domain of f

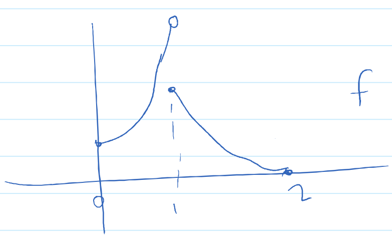
<u>Theorem</u> (Extreme Value Theorem)

If f is <u>continuous</u> on a <u>closed interval</u> $[a, b]$

then f attains an absolute maximum value $f(c)$
and an absolute minimum $f(d)$ for some c, d in $[a, b]$

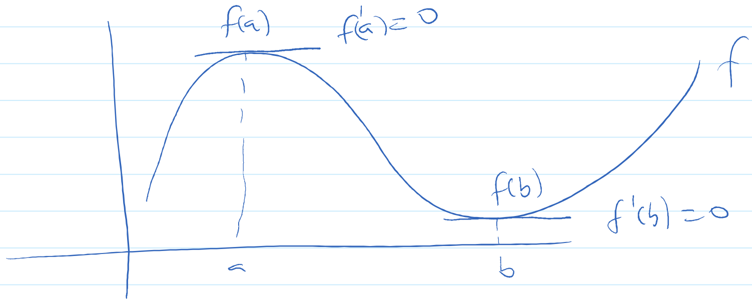
$f(x) = x^3$ on





<u>Fermat's Theorem</u> If f has a local maximum or minimum at c, and if $f'(c)$ exists

$\Downarrow$ then $f'(c) = 0$



f(a) is a local maximum

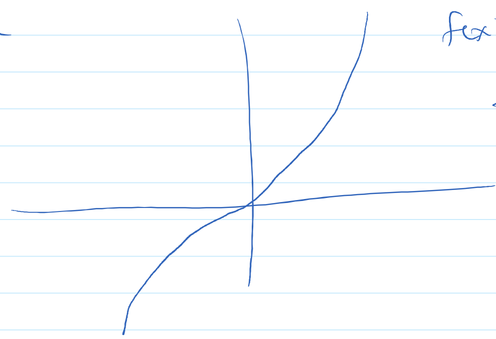
f(b) is a local minimum

Caution with using <u>fermat's theorem</u>

Even if $f'(c) = 0$ for some c in the domain of f there may not be a local maximum or minimum at c

The <u>converse</u> of fermat's Theorem is not true in general

Example

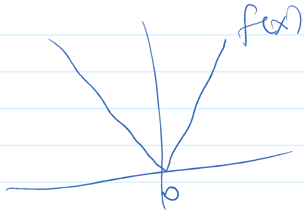


$f(x) = x^3$

$f'(x) = 3x^2$

$f'(0) = 0$

but we established that $f(0)$ is not a local maximum or local minimum for $f = x^3$



$f(x) = |x|$

$f'(0)$ does <u>not exist</u>

<u>Definition</u>

A critical value/number of a function is a number c in the domain of f such that either $f'(c) = 0$ or $f'(c)$ does not exist

I encourage that you look up this material in your text book