3.6 (cont'd)

- Derivative of logarithmic functions, ✓
- Using logarithmic properties in derivative problems ✓
- Derivative of inverse trigonometric functions

The Euler number 'e' as a limit

$$f(x) = \ln(x)$$

$$f'(x) = \frac{1}{x}, \quad f'(1) = 1$$

Let us use the derivative definition to compute $f'(1)$

$$f(x) = \ln(x), \quad f(1+x) = \ln(1+x)$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = \lim_{x \to 0} \frac{f(1+x) - f(1)}{x}$$

$$= \lim_{x \to 0} \frac{\ln(1+x) - \ln(1)}{x}$$

$$= \lim_{x \to 0} \frac{1}{x} \ln(1+x)$$

$$f'(1) = \lim_{x \to 0} \ln(1+x)^{1/x}$$

Since $f'(1) = 1$

So $$1 = \lim_{x \to 0} \ln(1+x)^{1/x}$$

make both sides exponent of e

$$e^1 = e^{\lim_{x \to 0} \ln(1+x)^{1/x}}$$

$$e = \lim_{x \to 0} e^{\ln(1+x)^{1/x}}$$

Aside

$$e^{\ln(x)} = x$$

Ph.D Leonhard

$1/x$

$$e = \lim_{x \to 0} e$$

$e = x$

Ph.D Leonhard Euler

$$\boxed{e = \lim_{x \to 0} (1+x)^{1/x}}$$

Sub $n = \frac{1}{x}$ if $x \to 0$, $n \to \infty$

$x = \frac{1}{n}$

$$\boxed{e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n} \quad (*)$$

Crash course on Compound Interest

$$\boxed{A = P\left(1 + \frac{r}{n}\right)^{nt}}$$

$n = 1$ (annual compounding)

$n = 2$ (semi-annual

$n = 4$ (quarterly

$n = 12$ (monthly

$\vdots$

$\Updownarrow$

$$\boxed{A = Pe^{rt}} \xrightarrow{n \Rightarrow \infty} \text{(Continuous) Compounding}$$

Continuous compounding

$n \to \infty$

Set $k = \frac{n}{r}$, $n = kr$

$$A = \lim_{n \to \infty} P\left(1 + \frac{r}{n}\right)^{nt} = \lim_{k \to \infty} P\left(1 + \frac{1}{k}\right)^{krt}$$

$$= \lim_{k \to \infty} P\left[\left(1 + \frac{1}{k}\right)^k\right]^{rt}$$

$$= P\left[\lim_{k \to \infty}\left(1 + \frac{1}{k}\right)^k\right]^{rt}$$

3.8

$$\boxed{A = Pe^{rt}}$$ continuous compounding

Aside

$$e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$$

$k \to \infty$, $n \to \infty$

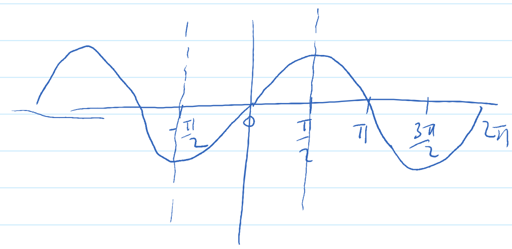
$k \to \infty, \; n \to \infty$

Derivatives of Inverse Trigonometric functions

$\sin^{-1}(x) \neq \dfrac{1}{\sin(x)}$

Consider

$$y = \sin^{-1}(x) \quad , \quad -\frac{\pi}{2} < y < \frac{\pi}{2} \; ✓$$



if we take sin of both sides

$$\sin(y) = \sin\left(\sin^{-1}(x)\right)$$

$$\sin(y) = x$$

$f\left(f^{-1}(x)\right) = x$

$\left(f \circ f^{-1}\right)(x) = x$

differentiate $\sin(y) = x$ implicitly

$$\frac{d}{dx}\sin(y) = \frac{d}{dx}(x)$$

$$\frac{d}{dy}(\sin(y)) \cdot \frac{dy}{dx} = 1$$

$$\cos(y) \cdot \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{\cos(y)} = \frac{1}{\sqrt{1-x^2}}$$

Aside

$\frac{d}{dx}(\sin(x)) = \cos(x)$

$\frac{d}{dy}(\sin(y)) = \cos(y)$

Aside

$\boxed{\sin^2(y) + \cos^2(y) = 1}$

$\cos^2(y) = 1 - \sin^2(y)$

$\cos(y) = \sqrt{1 - \sin^2(y)}$

$= \sqrt{1 - (\sin y)^2}$

$= \sqrt{1-x^2}$

$$\boxed{\text{differentiate } y = \sin^{-1}(x) \qquad \frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}} \; (*)$$

Take home Quiz

submit on or before mon

verify

① $\dfrac{d}{dx}\left(\sin^{-1}(x)\right) = \dfrac{1}{\sqrt{1-x^2}}$ (see today's note, equation (*))

② $\dfrac{d}{dx}\left(\cos^{-1}(x)\right) = -\dfrac{1}{\ldots}$ ( )

② $\frac{d}{dx}\left(\cos^{-1}(x)\right) = -\frac{1}{\sqrt{1-x^2}}$

(Donkeywork)

③ $\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2}$

This 30% of Exam #3

④ $\frac{d}{dx}\left(\csc^{-1}(x)\right) = -\frac{1}{x\sqrt{x^2-1}}$

⑤ $\frac{d}{dx}\left(\sec^{-1}(x)\right) = \frac{1}{x\sqrt{x^2-1}}$

⑥ $\frac{d}{dx}\left(\cot^{-1}(x)\right) = -\frac{1}{1+x^2}$

I will open dropbox on D2L, submit to the dropbox