Wednesday, October 7, 2020 7:58 AM

3.4 Chain Rule

$$y = F(x) = (f \circ g)(x) = f(g(x))$$

Approach 1

$$F'(x) = f'(g(x))\, g'(x)$$

Approach 2

$$y = F(x) = f(g(x)), \quad u = g(x)$$

$$f(u)$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Exam #2 practice

#82. Find the derivative of the function

$$F(x) = \frac{x^4 - 7x^3 + \sqrt{x}}{x^2}$$

use Quotient Rule

$$\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$$

$$F'(x) = \frac{x^2 \frac{d}{dx}(x^4 - 7x^3 + \sqrt{x}) - (x^4 - 7x^3 + \sqrt{x})\frac{d}{dx}(x^2)}{(x^2)^2}$$

$$= \frac{x^2\left(4x^3 - 21x^2 + \frac{1}{2\sqrt{x}}\right) - (x^4 - 7x^3 + \sqrt{x})(2x)}{x^4}$$

$$\frac{d}{dx}(\sqrt{x}) = \frac{d}{dx}(x^{1/2})$$

$$= \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2}x^{-1/2}$$

$$= \frac{1}{2x^{1/2}} = \frac{1}{2\sqrt{x}}$$

$$= \frac{4x^5 - 21x^4 + \frac{x^2}{2\sqrt{x}} - (2x^5 - 14x^4 + 2x\sqrt{x})}{x^4}$$

$$= \frac{\cancel{4x^5} - \cancel{21x^4} + \frac{x^2}{2\sqrt{x}} - \cancel{2x^5} + \cancel{14x^4} - 2x\sqrt{x}}{}$$

$$= \frac{\cancel{4x^5} - \cancel{21x^4} + \frac{x^2}{2\sqrt{x}} - \cancel{2x^5} + \cancel{14x^4} - 2x\sqrt{x}}{x^4}$$

$$= \boxed{\frac{2x^5 - 7x^4 + \frac{x^2}{2\sqrt{x}} - 2x\sqrt{x}}{x^4}}$$

$$= \frac{2x^{\cancel{5}}}{\cancel{x^4}} - \frac{7\cancel{x^4}}{\cancel{x^4}} + \frac{x^2}{2x^4\sqrt{x}} - \frac{2x\sqrt{x}}{x^4}$$

$$= 2x - 7 + \frac{1}{2x^2\sqrt{x}} - \frac{2\sqrt{x}}{x^3}$$

$$= 2x - 7 + \frac{1}{2x^{5/2}} - \frac{2}{x^{5/2}} \cdot \frac{2}{2}$$

$$= 2x - 7 + \frac{1}{2x^{5/2}} - \frac{4}{2x^{5/2}}$$

$$\boxed{f'(x) = 2x - 7 - \frac{3}{2x^{5/2}}}$$

$$x^2\sqrt{x} = x^2 \cdot x^{\frac{1}{2}}$$
$$= x^{2+\frac{1}{2}}$$
$$= x^{5/2}$$

$$\frac{2\sqrt{x}}{x^3} \cdot \frac{\sqrt{x}}{\sqrt{x}} = \frac{2\cancel{x}}{x^{\cancel{3}}\sqrt{x}}$$
$$= \frac{2}{x^2\sqrt{x}}$$
$$= \frac{2}{x^{5/2}}$$

#19 use a graph of

(a) $f(x) = \left(1 - \frac{3}{x}\right)^x$ to estimate value of $\lim_{x\to\infty} f(x) = \boxed{p}$

(b)

x	$f(x)$
10,000	x_1
100,000	x_2
1,000,000	x_3

$$\downarrow$$
$$\boxed{p}$$

$$\left(1 - \frac{3}{10{,}000}\right)^{10000} = x_1$$

$$\left(1 - \frac{3}{100{,}000}\right)^{100{,}000} = x_2$$

$$\left(1 - \frac{3}{1{,}000{,}000}\right)^{1{,}000{,}000} = x_3$$

#37. Consider the ffg

$g(\theta) = \theta - \cos(\theta)$

Recall

$\frac{d}{dx}(x) = 1$

$g(\theta) = \theta - \cos(\theta)$

$h(\theta) = \sin(\theta)$

Find $g'(\theta) = \frac{d}{d\theta}(g(\theta)) = \frac{d}{d\theta}(\theta - \cos(\theta))$

$= \frac{d}{d\theta}(\theta) - \frac{d}{d\theta}(\cos(\theta))$

$= 1 - (-\sin(\theta))$

$= 1 + \sin(\theta)$

find $h'(\theta) = \frac{d}{d\theta}(h(\theta)) = \frac{d}{d\theta}(\sin(\theta))$

$= \cos(\theta)$

$\frac{d}{dx}(x) = 1$

$\frac{d}{d\theta}(\theta) = 1$

$\frac{d}{dx}(\cos(x)) = -\sin(x)$

$\frac{d}{d\theta}(\cos\theta) = -\sin(\theta)$

$\frac{d}{dx}(\sin(x)) = \cos(x)$

$\frac{d}{d\theta}(\sin(\theta)) = \cos(\theta)$

Product Rule

$(fg)' = fg' + gf'$

Differentiate

$f(\theta) = (\theta - \cos(\theta))\sin(\theta)$

$f'(\theta) = (\theta - \cos(\theta))\frac{d}{d\theta}(\sin\theta) + \sin(\theta)\cdot\frac{d}{d\theta}(\theta - \cos(\theta))$

$= (\theta - \cos(\theta))\cos(\theta) + \sin(\theta)(1 + \sin(\theta))$

$= \theta\cos(\theta) - \cos^2(\theta) + \sin(\theta) + \sin^2(\theta)$

A very good Exercise

#11. Find the Limit

$$\lim_{x\to\infty}\frac{\sqrt{1+25x^6}}{4-x^3} = \lim_{x\to\infty}\frac{\frac{\sqrt{1+25x^6}}{x^3}}{\frac{4-x^3}{x^3}}$$

$$= \lim_{x\to\infty}\frac{\frac{\sqrt{1+25x^6}}{\sqrt{x^6}}}{\frac{4}{x^3}-\frac{x^3}{x^3}}$$

$$= \lim_{x\to\infty}\frac{\sqrt{\frac{1+25x^6}{x^6}}}{\frac{4}{x^3}-1}$$

$x^3 = x^{6/2}$

$= (x^6)^{1/2}$

$= \sqrt{x^6}$

$\lim \sqrt{1 + 25x^6}$

$$\lim_{x\to\infty} \frac{\sqrt{\frac{}{x^6}+\frac{}{x^6}}}{\frac{4}{x^3}-1}$$

$$= \lim_{x\to\infty} \frac{\sqrt{\frac{1}{x^6}+25}}{\frac{4}{x^3}-1}$$

$$\approx \frac{\sqrt{\frac{1}{\infty}+25}}{\frac{4}{\infty}-1} = \frac{\sqrt{0+25}}{0-1} = \frac{5}{-1}$$

$$= \boxed{-5}$$

#30 Differentiate

$$P(w) = \frac{8w^2-4w+2}{\sqrt{w}}$$

Quotient Rule

$$\left(\frac{f}{g}\right)' = \frac{gf'-fg'}{g^2}$$

$$P'(w) = \frac{\sqrt{w}\cdot\frac{d}{dw}(8w^2-4w+2) - (8w^2-4w+2)\frac{d}{dw}(\sqrt{w})}{(\sqrt{w})^2}$$

$$= \frac{\sqrt{w}\,(16w-4+0) - (8w^2-4w+2)\left(\frac{1}{2\sqrt{w}}\right)}{w}$$

$$= \frac{16w\sqrt{w} - 4\sqrt{w} - \left(\frac{4w^2}{\sqrt{w}} - \frac{2w}{\sqrt{w}} + \frac{1}{\sqrt{w}}\right)}{w}$$

$$= \frac{16\cancel{w}\sqrt{w}}{\cancel{w}} - \frac{4\sqrt{w}}{w} - \frac{4w^{\cancel{2}}}{\cancel{w}\sqrt{w}} + \frac{2\cancel{w}}{\cancel{w}\sqrt{w}} - \frac{w}{\sqrt{w}}$$

$$= 16\sqrt{w} - \frac{4\sqrt{w}}{w} - \frac{4w}{\sqrt{w}} + \frac{2}{\sqrt{w}} - \frac{w}{\sqrt{w}}$$

$$= 16\sqrt{w} - \frac{4}{\sqrt{w}} + \frac{2}{\sqrt{w}} - \frac{4w}{\sqrt{w}} - \frac{w}{\sqrt{w}}$$

$$= 16\sqrt{w} - \frac{2}{\sqrt{w}} - \frac{5w}{\sqrt{w}}$$

$$\frac{d}{dw}(\sqrt{w}) = \frac{d}{dw}(w^{1/2})$$

$$= \frac{1}{2}w^{-\frac{1}{2}}$$

$$= \frac{1}{2w^{1/2}}$$

$$= \frac{1}{2\sqrt{w}}$$

$$(\sqrt{w})^2 = (w^{1/2})^2$$

$$= w$$

$$\frac{4\sqrt{w}}{w}\cdot\frac{\sqrt{w}}{\sqrt{w}} = \frac{4\cancel{w}}{\cancel{w}\sqrt{w}}$$