Monday, October 5, 2020 7:59 AM

3.3 Derivatives of Trigonometric Functions

1. $\frac{d}{dx}\left(\sin(x)\right) = \cos(x)$
2. $\frac{d}{dx}\left(\cos(x)\right) = -\sin(x)$
3. $\frac{d}{dx}\left(\tan(x)\right) = \sec^2(x)$
4. $\frac{d}{dx}\left(\csc(x)\right) = -\csc(x)\cdot\cot(x)$
5. $\frac{d}{dx}\left(\sec(x)\right) = \sec(x)\cdot\tan(x)$
6. $\frac{d}{dx}\left(\cot(x)\right) = -\csc^2(x)$

Where

1. $\sec(x) = \frac{1}{\cos(x)}$
2. $\csc(x) = \frac{1}{\sin(x)}$
3. $\cot(x) = \frac{1}{\tan(x)}$

$\sec^2(x) = \left[\sec(x)\right]^2$

$\csc^2(x) = \left[\csc(x)\right]^2$

$\sin^2(x) = \left[\sin(x)\right]^2$

⋮

Recall $\boxed{\lim_{h\to 0}\frac{\sin(h)}{h} = 1}$ ✓ $\boxed{\lim_{h\to 0}\frac{\cos(h)-1}{h} = 0}$

Exercise

$\boxed{y = x^2\sin(x)}$, differentiate y

$\frac{d}{dx}(y) = \frac{d}{dx}\left(x^2\sin(x)\right)$

$\frac{dy}{dx} = \frac{d}{dx}\left(x^2\sin(x)\right)$

$= x^2\frac{d}{dx}(\sin(x)) + \sin(x)\frac{d}{dx}(x^2)$

$= x^2\cos(x) + \sin(x)\,2x$

$\boxed{\frac{dy}{dx} = x^2\cos(x) + 2x\sin(x)}$

$f(x) = x^2\sin(x)$

$\frac{d}{dx}(f(x)) = \frac{d}{dx}\left(x^2\sin(x)\right)$

Product rule

$(fg)' = fg' + gf'$

$\frac{d}{dx}(fg) = f\frac{d}{dx}(g) + g\frac{d}{dx}(f)$

Exercise 1

Quotient rule

$$\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$$

$$\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{g\frac{d}{dx}(f) - f\frac{d}{dx}(g)}{g^2}$$

(5) $y = \dfrac{\sec(x)}{1+\tan(x)}$

$$\frac{dy}{dx} = \frac{d}{dx}\left[\frac{\sec(x)}{1+\tan(x)}\right]$$

$$= \frac{(1+\tan(x))\frac{d}{dx}(\sec(x)) - \sec(x)\frac{d}{dx}(1+\tan(x))}{(1+\tan(x))^2}$$

$$= \frac{(1+\tan(x))(\sec(x)\cdot\tan(x)) - \sec(x)(0+\sec^2(x))}{(1+\tan(x))^2}$$

$$= \frac{\sec(x)\tan(x) + \sec(x)\tan^2(x) - \sec^3(x)}{(1+\tan(x))^2}$$

$$= \frac{\sec(x)\left[\tan(x) + \tan^2(x) - \sec^2(x)\right]}{(1+\tan(x))^2}$$

$$= \frac{\sec(x)\left[\tan(x) + \tan^2(x) - (1+\tan^2(x))\right]}{(1+\tan(x))^2}$$

$$= \frac{\sec(x)\left[\tan(x) + \cancel{\tan^2(x)} - 1 - \cancel{\tan^2(x)}\right]}{\left[1+\tan(x)\right]^2}$$

$$= \frac{\sec(x)(\tan(x)-1)}{(1+\tan(x))^2}$$

trig Identities

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

$A = B$

$\cos(A-A) = \cos A\cos A + \sin A\sin A$

$\cos(0) = \cos^2 A + \sin^2 A$

$1 = \cos^2 A + \sin^2 A$

$$\frac{1}{\cos^2 A} = \frac{\cos^2 A + \sin^2 A}{\cos^2 A}$$

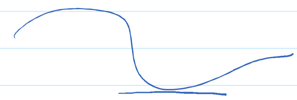
$$\frac{1}{\cos^2 A} = \frac{\cos^2 A}{\cos^2 A} + \frac{\sin^2 A}{\cos^2 A}$$

$\sec^2 A = 1 + \tan^2 A$

(5) [For what values of x does the graph of y have an horizontal tangent?]

$\Rightarrow$ find the values of x when

$$f'(x) = 0$$



find x, when

$$\frac{\sec(x)\,(\tan x - 1)}{(1 + \tan(x))^2} = 0$$

$\Rightarrow \quad \sec(x)\,(\tan(x) - 1) = 0$

Recall zeroth property

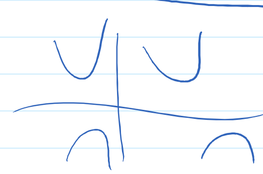
$a \cdot b = 0$

either $a = 0$ or $b = 0$

or both

How convince yourselves that $\sec(x)$ is never 0

How? (graph $\sec(x)$ with desmos)



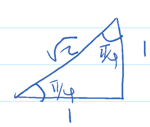
So $\tan(x) - 1 = 0$

$\tan(x) = 1$

$x = \arctan(1)$

$$\boxed{x = \frac{\pi}{4} + n\pi \qquad n = 0, 1, 2, \ldots}$$

$\tan(x) = 1$



$\tan\left(\frac{\pi}{4}\right) = 1$

Prove that

$$\lim_{h\to 0} \frac{\cos(h)-1}{h} = 0$$

Use the fact we have proved

$$\lim_{h\to 0} \frac{\sin(h)}{h} = 1$$

Pf

$$\lim_{h\to 0} \frac{\cos(h)-1}{h} \cdot \frac{\cos(h)+1}{\cos(h)+1}$$

$$= \lim_{h\to 0} \frac{\cos^2(h)-1}{h(\cos(h)+1)}$$

$$= \lim_{h\to 0} \frac{-\sin^2(h)}{h(\cos(h)+1)}$$

$$= -\lim_{h\to 0} \left(\frac{\sin(h)}{h} \cdot \frac{\sin(h)}{\cos(h)+1} \right)$$

$$= -\lim_{h\to 0}\left(\frac{\sin(h)}{h}\right) \cdot \lim_{h\to 0}\left(\frac{\sin(h)}{\cos(h)+1}\right)$$

$$= -1 \cdot \frac{\sin(0)}{\cos(0)+1}$$

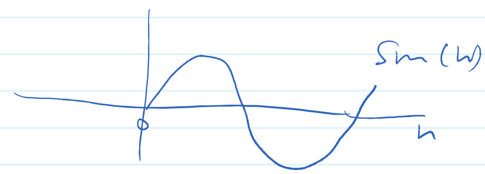
$$= -1 \cdot \frac{0}{1+1}$$

$$= -1 \cdot 0 = 0$$

Recall

$$1 = \cos^2(h) + \sin^2(h)$$

$$-\sin^2(h) = \cos^2(h) - 1$$



$\sin(0) = 0$

$\cos(0) = 1$

Exercise use $\boxed{\lim_{h\to 0} \frac{\sin(h)}{h} = 1}$ $\boxed{\lim_{x\to 0} \frac{\sin(x)}{x} = 1}$

Show that $\boxed{\lim_{x\to 0} \frac{\sin 7x}{4x} = \frac{7}{4}}$

$$= \lim_{x\to 0} \frac{7}{4} \cdot \frac{\sin(7x)}{7x} \quad \text{———} \quad (*)$$

If we let $h = 7x$

when $h \to 0$ then $x \to 0$

So $(*)$ becomes

So (*) becomes

$$\frac{7}{4}\left(\lim_{x\to 0}\frac{\sin(7x)}{7x}\right) = \frac{7}{4}\left(\lim_{h\to 0}\frac{\sin(h)}{h}\right)$$

$$= \frac{7}{4}(1)$$

$$= \frac{7}{4}$$