Chapter 3 : Differentiation Rules

✓3.1 : Derivatives of Polynomials and Exponential functions

✓3.2 : Product Rule and Quotient Rule

3.3 : Derivatives of Trigonometric functions

3.2 Product Rule

If f, g are both differentiable functions

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$ Leibniz

$$(fg)' = fg' + gf'$$

Examples

If $\boxed{f(x) = xe^x}$, find $\boxed{f'(x)}$

a) $\frac{d}{dx}[xe^x] = x\frac{d}{dx}[e^x] + e^x\frac{d}{dx}[x]$

$= xe^x + e^x \cdot 1$

$\boxed{f'(x) = e^x(x+1)}$

$f'(x) = e^x(x+1)$ (first derivative)

f is given function
f' (first derivative)
f'' (second derivative)
f''' (third derivative)
$\vdots$
$f^{(n)}$ (nth derivative)

find f'' (second derivative)

b) $f''(x) = \frac{d}{dx}[f'(x)] = \frac{d}{dx}[e^x(x+1)] = e^x\frac{d}{dx}[x+1] + (x+1)\frac{d}{dx}[e^x]$

$= e^x \cdot 1 + (x+1)e^x$

$= e^x(1 + x + 1)$

$= e^x(x+2)$

(find the third derivative)

c) $f'''(x) = \frac{d}{dx}[f''(x)] = \frac{d}{dx}[e^x(x+2)] = e^x\frac{d}{dx}[x+2] + (x+2)\frac{d}{dx}[e^x]$

$= e^x \cdot 1 + (x+2)e^x$

$= e^x(1 + x+2)$

$= e^x(x+3)$

If $\boxed{f(x) = e^x x}$, we see that

$f'(x) = e^x(x+1)$ first derivative

$f''(x) = e^x(x+2)$ second derivative

$f'''(x) = e^x(x+3)$ third derivative

$f^{(4)}(x) = e^x(x+4)$ fourth derivative

$\vdots$

$f^{(n)}(x) = e^x(x+n)$ nth derivative

Remark (all 1 - F20 class)

If $f(x) = e^x x$, then the nth derivative for $n \geq 1$ is given by $f^{(n)} = e^x(x+n)$

From the Homework

$f(x) = x^n$, find $f', f'', f''', \ldots, f^{(n)}$

$f'(x) = n x^{n-1}$

$f''(x) = n(n-1)x^{n-1-1} = n(n-1)x^{n-2}$

$f'''(x) = n(n-1)(n-2)x^{n-3}$

$\vdots$

$n! = n(n-1)(n-2)(n-3)\cdots 3 2 \cdot 1$

$f^{(n)}(x) = n(n-1)(n-2)(n-3)\cdots x^{n-n}$

$= n(n-1)(n-2)(n-3)\cdots x^0$

$= n(n-1)(n-2)(n-3)\cdots 2 \cdot 1 = n!$

Remark

if $f(x) = x^n$, then $f^{(n)}(x) = n!$

Quotient Rule

If f, g are both differentiable, then

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{g(x)^2}$$ Leibniz

$$\boxed{\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}}$$

Exercise

$f(x) = \dfrac{x^2+x-2}{x^3+6}$, find $f'(x)$

$$\frac{d}{dx}\left[\frac{x^2+x-2}{x^3+6}\right] = \frac{(x^3+6)\frac{d}{dx}[x^2+x-2] - (x^2+x-2)\frac{d}{dx}[x^3+6]}{(x^3+6)^2}$$

$$= \frac{(x^3+6)(2x+1-0) - (x^2+x-2)(3x^2+0)}{(x^3+6)^2}$$

$$= \frac{(x^3+6)(2x+1) - (x^2+x-2)3x^2}{(x^3+6)^2}$$

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Integer exponents

$$\boxed{\begin{aligned} &x^3 \cdot 2x \\ &2 \cdot x^3 \cdot x^1 \\ &2 \cdot x^{3+1} \end{aligned}}$$

$$= \frac{2x^4 + x^3 + 12x + 6 - (3x^4 + 3x^3 - 6x^2)}{(x^3+6)^2}$$

$$= \frac{2x^4 + x^3 + 12x + 6 - 3x^4 - 3x^3 + 6x^2}{(x^3+6)^2}$$

$$= \frac{-x^4 - 2x^3 + 6x^2 + 12x + 6}{(x^3+6)^2}$$

Product Rule

f, g are differentiable

$$(fg)' = fg' + gf'$$

Quotient Rule

f, g are differentiable

$$\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$$

Exercise

$$f(t) = \sqrt{t}\,(a+bt) = t^{1/2}(a+bt)$$

$$f'(t) = \frac{d}{dt}\left[t^{1/2}(a+bt)\right]$$

$$= t^{1/2}\frac{d}{dt}[a+bt] + (a+bt)\frac{d}{dt}\left[t^{1/2}\right]$$

$$= t^{1/2}(0+b) + (a+bt)\,\frac{1}{2}t^{\frac{1}{2}-1}$$

$$= b\sqrt{t} + (a+bt)\,\frac{1}{2}t^{-1/2}$$

$$= b\sqrt{t} + \frac{a+bt}{2t^{1/2}}$$

$$\boxed{f'(t) = b\sqrt{t} + \frac{(a+bt)}{2\sqrt{t}}}$$

Radical Notation	&	Rational Exponent
$\sqrt{a}$	=	$a^{1/2}$
$\sqrt[3]{a}$	=	$a^{1/3}$
$\sqrt[4]{a}$	=	$a^{1/4}$
$\vdots$		
$\sqrt[n]{a}$	=	$a^{1/n}$
$\sqrt[n]{a^m}$	=	$a^{\frac{m}{n}}$
$\left(\sqrt[n]{a}\right)^m$	=	$a^{\frac{m}{n}}$
$t^{-\frac{1}{2}}$	=	$\frac{1}{t^{1/2}}$

New set of HW on webassign

3.1, 3.2, $\boxed{3.3}$

Exam 2 practice (set of new exercises)

Exam 2

2.5, 2.6, 2.7, 2.8, 3.1, 3.2, $\boxed{3.3}$

Aleks

when you took the assessment on Aleks

learning modules