2.6, 2.7, 2.8

return for HW — 2.6

complete — 2.7

start — 2.8

Review ε-δ, (Sometimes there is an N)

① $\lim_{x \to a} f(x) = L$

ε-δ

given $\varepsilon > 0$ there exist a $\delta > 0$ such that

if $0 < |x-a| < \delta$ then $|f(x) - L| < \varepsilon$

② $\lim_{x \to \infty} f(x) = L$

ε-N

given $\varepsilon > 0$ there exist an N such that

If $x > N$ then $|f(x) - L| < \varepsilon$

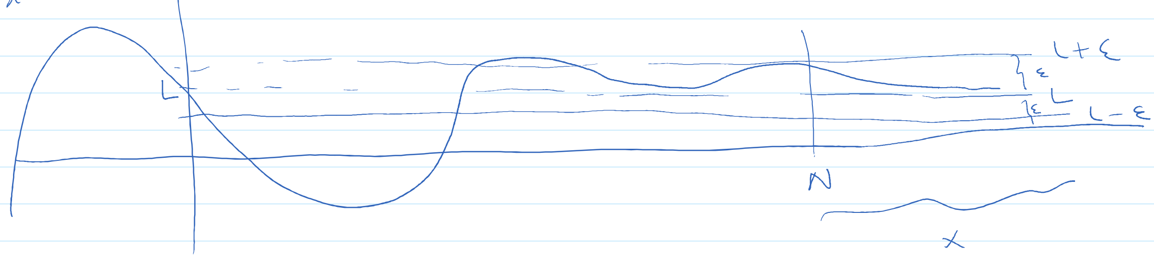
③ $\lim_{x \to -\infty} f(x) = L$

ε-N

given $\varepsilon > 0$ there exist an N such that

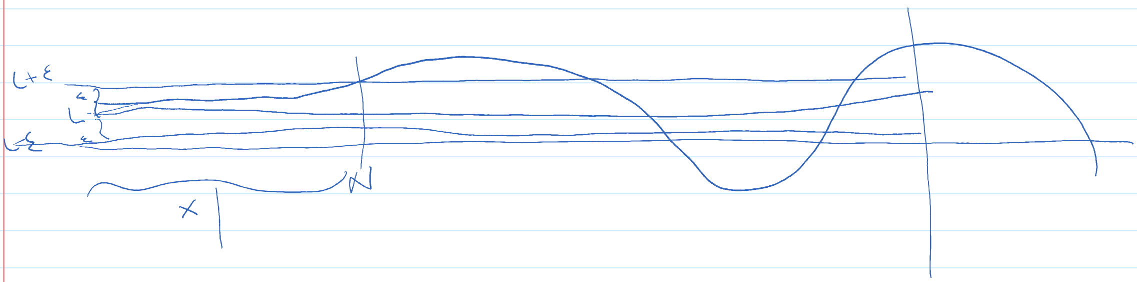
If $x < N$ then $|f(x) - L| < \varepsilon$

Sketch for ②



If we make ε smaller you need a bigger N

sketch for (3)



If we make ε smaller you need a smaller N

use ε - N (definition)

show $\lim_{x\to\infty} \dfrac{3x^2-x-2}{5x^2+4x+1} = \dfrac{3}{5} = 0.6$ $\boxed{\text{given } \varepsilon = 0.1}$

Pf

given $\varepsilon > 0$ $\boxed{\text{find } N}$ such that

If $x > N$ then $\left|\dfrac{3x^2-x-2}{5x^2+4x+1} - 0.6\right| < 0.1$

$$-0.1 < \frac{3x^2-x-2}{5x^2+4x+1} - 0.6 < 0.1$$

$$-0.1+0.6 < \frac{3x^2-x-2}{5x^2+4x+1} < 0.1+0.6$$

$$0.5 < \frac{3x^2-x-2}{5x^2+4x+1} < 0.7$$

find x such that the function lies between 0.5 and 0.7

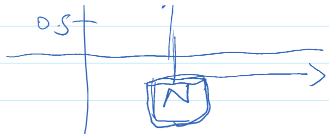
using a graphing calculator (desmos)

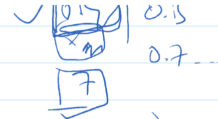
$x > 7$ we see $0.5 < \dfrac{3x^2-x-2}{5x^2+4x+1} < 0.7$

$\boxed{N = 7}$

x	f(x)
x	0.49
6	0.48
6.2	0.49
0.5	0.5
x = N	0.7 ...
7	

$\boxed{N = 1}$





Exercise

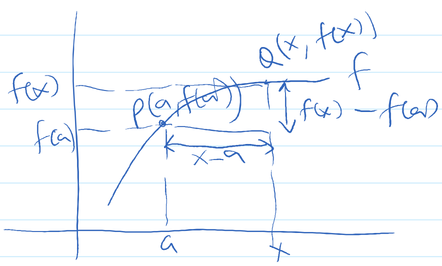
Find an N when $\underline{\varepsilon = 0.01}$ (give me feedback tomorrow)

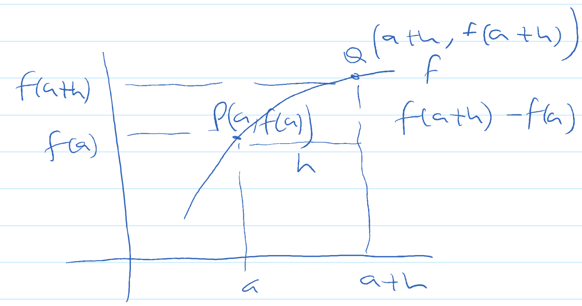
2.7 Derivatives and Rates of Change

Definition

The tangent line to the curve $y = f(x)$ at the point $P(a, f(a))$ is the line through P with slope

$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \qquad m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$





$h = x - a$

$a + h = x$

Exercise

Find the equation of the tangent line to the hyperbola $f(x) = y = \frac{3}{x}$ at the point $(3, 1)$

$a = 3$

$f(a) = f(3) = \frac{3}{3} = 1$

$f(a+h) = f(3+h) = \frac{3}{3+h}$

Solution

$$m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{3}{3+h} - 1}{h}$$

$$= \lim_{h \to 0} \frac{\frac{3-(3+h)}{3+h}}{h}$$

$$= \lim_{h \to 0} \frac{3-(3+h)}{h(3+h)}$$

$$= \lim_{h \to 0} \frac{\cancel{3}-\cancel{3}-h}{h(3+h)}$$

$$= \lim_{h \to 0} \frac{-h}{h(3+h)}$$

$$m = \lim_{h \to 0} \frac{-1}{3+h} = -\frac{1}{3}$$

find the equation of the tangent line

Point-slope form $\quad y - y_1 = m(x - x_1)$

given $\quad (x_1, y_1) = (3, 1)$

$$m = -\frac{1}{3}$$

$$y - 1 = -\frac{1}{3}(x - 3)$$

$$3y - 3 = -x + 3$$

$$3y + x - 6 = 0$$

Definition

Derivative of a function f at a point 'a'

① $$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

② $$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

Exercise

to show that

use definition (2) above to show that

the derivative $f(x) = \frac{1}{\sqrt{x}}$ at a point 'a'

is $f'(a) = -\frac{1}{2a^{3/2}}$

$f(x) = \frac{1}{\sqrt{x}}$

$f(a) = \frac{1}{\sqrt{a}}$

$\frac{1}{\sqrt{x}} - \frac{1}{\sqrt{a}}$

$\frac{\sqrt{a} - \sqrt{x}}{\sqrt{x}\sqrt{a}}$

Solution

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{\frac{1}{\sqrt{x}} - \frac{1}{\sqrt{a}}}{x - a}$$

$$= \lim_{x \to a} \frac{\sqrt{a} - \sqrt{x}}{\sqrt{x}\sqrt{a}\,(x-a)}$$

$$= \lim_{x \to a} \frac{\sqrt{a} - \sqrt{x}}{\sqrt{ax}\,(x-a)} \cdot \frac{\sqrt{a} + \sqrt{x}}{\sqrt{a} + \sqrt{x}}$$

$$= \lim_{x \to a} \frac{a - x}{\sqrt{ax}\,(x-a)\,(\sqrt{a} + \sqrt{x})}$$

$$= \lim_{x \to a} \frac{-\cancel{(x-a)}}{\sqrt{ax}\,\cancel{(x-a)}\,(\sqrt{a} + \sqrt{x})}$$

$$= \lim_{x \to a} \frac{-1}{\sqrt{ax}\,(\sqrt{a} + \sqrt{x})}$$

$$= \frac{-1}{\sqrt{a \cdot a}\,(\sqrt{a} + \sqrt{a})}$$

$$= \frac{-1}{a \cdot 2\sqrt{a}}$$

$$= \frac{-1}{2a^1 \cdot a^{1/2}}$$

$$= \frac{-1}{2a^{1+1/2}}$$

$$\boxed{f'(a) = \frac{-1}{2a^{3/2}}}$$