Thursday, September 17, 2020 7:14 AM

Recall the following from 2.6

Limit at Infinity

$$\lim_{x \to \infty} f(x) = L$$

$\Downarrow$ the line $y = L$ is an Horizontal asymptote

Infinite Limit

$$\lim_{x \to a} f(x) = \infty$$

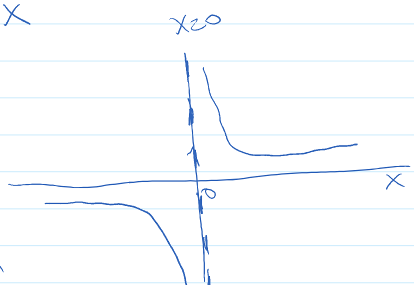
$\Downarrow$ the line $x = a$ is a vertical asymptote

Example

$f(x) = \frac{1}{x}$

$$\lim_{x \to 0^+} \frac{1}{x} = \infty$$

$\Downarrow$ line $x = 0$ is a vertical asymptote



Infinite Limit (DNE Limit)

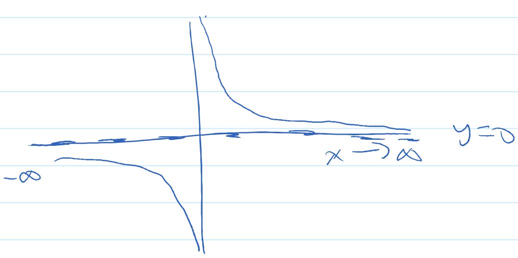
$$\lim_{x \to 0^-} \frac{1}{x} = -\infty$$

$\Downarrow$ line $x = 0$ is a vertical asymptote

$f(x) = \frac{1}{x}$

$$\lim_{x \to \infty} \frac{1}{x} = 0$$

$\Downarrow$ line $y = 0$ is an horizontal asymptote



$$\lim_{x \to -\infty} \frac{1}{x} = 0$$

$\Downarrow$ line $y = 0$ is an horizontal asymptote

Theorem

$$\lim_{x\to\infty} \frac{1}{x^r} = 0 \qquad (r>0)$$

Exercise

$$\lim_{x\to\infty} \frac{(3x^2-x-2)\frac{1}{x^2}}{(5x^2+4x+1)\frac{1}{x^2}} = \lim_{x\to\infty} \frac{\frac{3x^2-x-2}{x^2}}{\frac{5x^2+4x+1}{x^2}}$$

term with the highest degree in the denominator

$$\frac{x^1}{x^2} = \frac{1}{x^{2-1}} = \frac{1}{x}$$

$$= \lim_{x\to\infty} \frac{\frac{3x^2}{x^2} - \frac{x}{x^2} - \frac{2}{x^2}}{\frac{5x^2}{x^2} + \frac{4x}{x^2} + \frac{1}{x^2}}$$

$$= \lim_{x\to\infty} \frac{3 - \frac{1}{x} - \frac{2}{x^2}}{5 + \frac{4}{x} + \frac{1}{x^2}}$$

$$= \frac{\lim_{x\to\infty}\left(3 - \frac{1}{x} - \frac{2}{x^2}\right)}{\lim_{x\to\infty}\left(5 + \frac{4}{x} + \frac{1}{x^2}\right)}$$

Theorem

$$\lim_{x\to\infty} \frac{1}{x^r} = 0 \qquad (r>0)$$

rational number

$$= \frac{\lim_{x\to\infty} 3 - \lim_{x\to\infty}\frac{1}{x} - \lim_{x\to\infty}\frac{2}{x^2}}{\lim_{x\to\infty} 5 + \lim_{x\to\infty}\frac{4}{x} - \lim_{x\to\infty}\frac{1}{x^2}}$$

$$\lim_{x\to a} c = c$$

$$\lim_{x\to\infty} 3 = 3$$

$$\lim_{x\to\infty} 5 = 5$$

$$= \frac{\lim_{x\to\infty} 3 - \lim_{x\to\infty}\frac{1}{x} - 2\lim_{x\to\infty}\frac{1}{x^2}}{\lim_{x\to\infty} 5 + 4\lim_{x\to\infty}\frac{1}{x} - \lim_{x\to\infty}\frac{1}{x^2}}$$

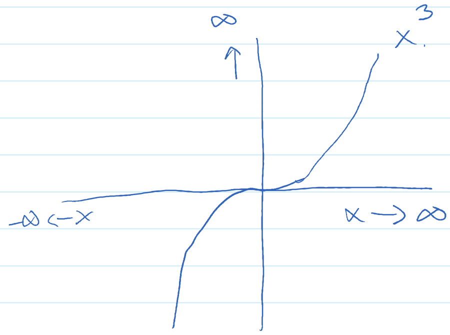
$$= \frac{3 - 0 - 2(0)}{5 + 4(0) - 0}$$

$$= \frac{3}{5}$$

Infinite Limit at Infinity

$$\lim_{x \to \infty} f(x) = \infty$$

$$\lim_{x \to \infty} x^3 = \infty$$



$$\lim_{x \to -\infty} = -\infty$$

$$\lim_{x \to \infty} e^x = \infty$$

End behavior of a function

How a function behaves as $x \to \infty$ and as $x \to -\infty$

Naive

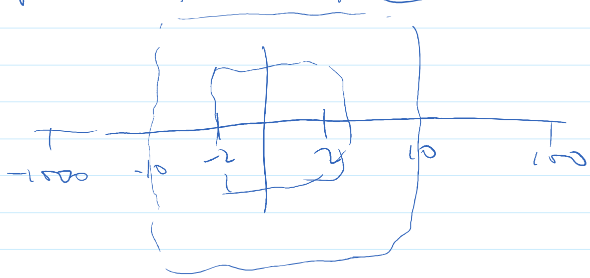
$f(x) = 3x^5$

$g(x) = 3x^5 - 5x^3 + 2x$

x-intercepts (zeros)

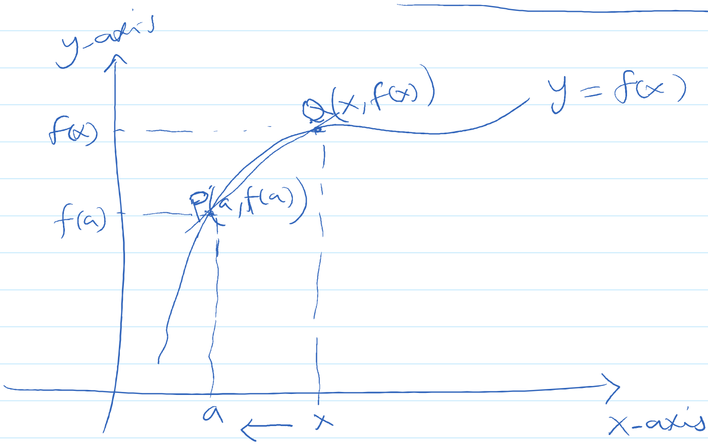
graph $f(x) = \boxed{3x^5}$, $g(x) = \boxed{3x^5} \underline{- 5x^3 + 2x}$

have similar behavior as $x \to \infty$, $x \to -\infty$



2.7 Derivatives and Rates of Change

2.7 Derivatives and Rates of change



Q
We want to find the tangent line to the curve $y = f(x)$ at the point P

Recall
our approach, is to pick a nearby point $Q(x, f(x))$ $(x \neq a)$
we can compute the slope

$$m_{PQ} = \frac{f(x) - f(a)}{x - a} \quad \left(\text{slope of secant line } PQ\right)$$

we let Q approach P along the curve, by letting x approach a

notice

$$m_{PQ} \longrightarrow m$$

m is the slope of the tangent line to the curve at the point P

Definition
The tangent line to the curve $y = f(x)$ at the point $P(a, f(a))$ is the line through P with slope

$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{x \to a} m_{PQ}$$

Example
Find the equation of the tangent line to the

parabola $y = x^2$ at the point $P(1,1)$

Solution

$$m = \lim_{x\to 1} \frac{f(x) - f(1)}{x-1} = \lim_{x\to 1} \frac{x^2 - 1}{x-1}$$

$$= \lim_{x\to 1} \frac{\cancel{(x-1)}(x+1)}{\cancel{x-1}}$$

$$= \lim_{x\to 1} x+1$$

$$= 1+1$$

$$= 2$$

difference of squares

$x^2 - 1$

$= x^2 - 1^2$

$= (x-1)(x+1)$

$$\boxed{a^2 - b^2 = (a-b)(a+b)}$$

To find find the equation of the tangent line, use the point-slope form where $(x_1, y_1) = (1,1)$

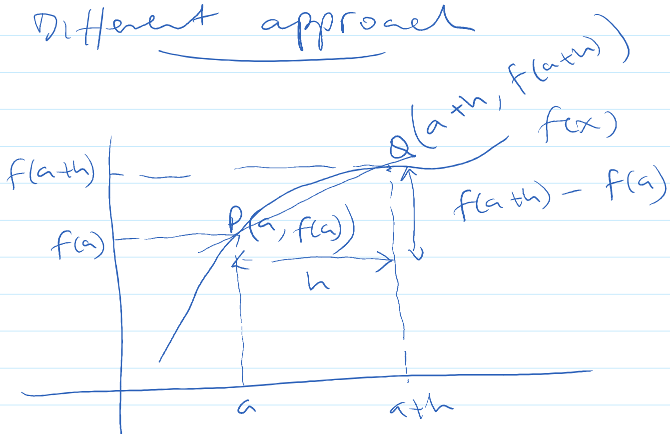
$$y - y_1 = m(x - x_1)$$

$$y - 1 = 2(x-1)$$

$$y = 2x - 1$$

$$m_{PQ} = \frac{f(x) - f(a)}{x-a}$$

Different approach



set $\boxed{x = a+h}$

$x - a = h$

$$m_{PQ} = \frac{f(a+h) - f(a)}{h}$$

$$\boxed{m = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}}$$

Derivatives

① The derivative of a function f at a point a denoted $f'(a)$ (read as f prime of a)

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

(ii) $$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Homework due on Saturday 09/19

2.5, 2.6 ✓