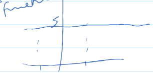
Exam 1 $\boxed{\text{30 problems}}$

✓ Functions (only prerequisite) < College Algebra (functions) / precalculus (Trig functions)

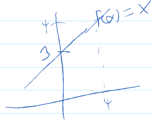
- 2.1
- 2.2
- 2.3
- 2.4

Linear functions

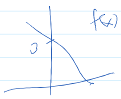
constant function

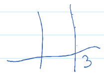


Example $f(x)=5$



$f(x)=-x+3$





not a function (It fails the vertical line test)

line $x=3$ (for every value of y, $x=3$)

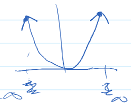
vertical line test

A vertical line intersect the graph of a function at exactly one point

Nonlinear functions

polynomial function

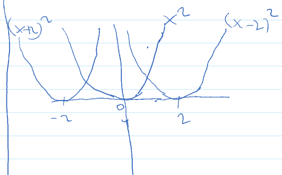
$f(x)=x^2$



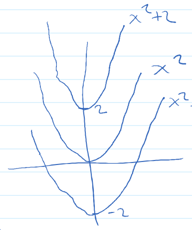
$g(x)=a(x-h)^2+k$

stretching or shrinking / horizontal shift / vertical shift

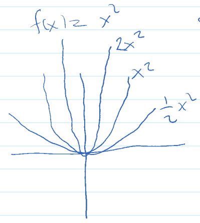
$f(x)=x^2$ $g(x)=(x-2)^2$



$f(x)=x^2$ $g(x)=x^2+2$



$f(x)=x^2$ $g(x)=2x^2$ $r(x)=\frac{1}{2}x^2$



Recall

$f(x)=x^2$

sketch $g(x)=2(x-2)^2+3$ (vertex form)

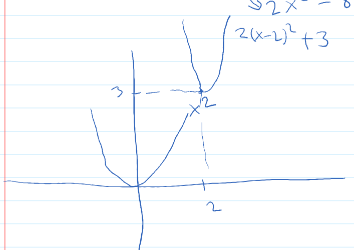
$2(x^2-4x+4)+3$

multiply

$2x^2-8x+8+3$

$2x^2-8x+11$

vertex formula



Arrogant Remark

There is only one quadratic function

$f(x)=x^2$ ✓

(any other quadratic function is a transformation of $f(x)=x^2$)

$g(x)=50x^2+1.5x+3$

Rational function

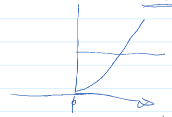
$f(x)=\frac{P(x)}{Q(x)}$, P, Q are polynomial functions ($Q(x)\neq 0$)

| make $f(x)=x^2$ 1-1 on $(0,\infty)$

Rational function

$f(x) = \frac{P(x)}{Q(x)}$, P, Q are polynomial functions $(Q(x) \neq 0)$

make $f(x) = x^2$ 1−1 on $(0, \infty)$



an horizontal line intersect 1−1 at exactly one point

Inverse functions

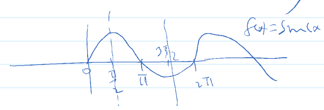
(not reciprocals)

Example

find the inverse function (if it exist)

$f(x) = 2x + 3$

(Inverse function exist only for 1−1)



1. pick a number x
2. multiply by 2
3. add 3

(But $f^{-1}(x) = \frac{1}{2x+3}$ ✗ reciprocal)

VAS method

| Ninja movies / Kung fu movies ✓

To find the inverse

Do reverse of the operations above

1. pick a number x
2. subtract 3
3. divide by 2

$f^{-1}(x) = \frac{x-3}{2}$

$\boxed{f(x) = \sqrt[3]{2x+3}}$

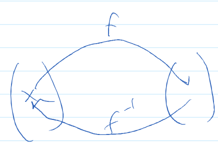
1. pick a number x
2. multiply by 2
3. add 3
4. take cube root

$(\sqrt[3]{x})^3 = (x^{1/3})^3 = x$

To find inverse of f

1. pick a number x
2. raise to power 3
2. subtract 3
3. divide by 2

$\boxed{f^{-1}(x) = \frac{x^3 - 3}{2}}$



$f \circ f^{-1}(x) = f^{-1} \circ f(x) = x$ ✓

Exercise

Show that f^{-1} of $\sqrt[3]{2x+3}$ is $\frac{x^3-3}{2}$

$$(f \circ f^{-1})(x) = f(f^{-1}(x))$$

$$= f\left(\frac{x^3-3}{2}\right) = \sqrt[3]{\cancel{2}\left(\frac{x^3-3}{\cancel{2}}\right) + 3}$$

$$= \sqrt[3]{x^3 \cancel{-3} \cancel{+3}}$$

$$= \sqrt[3]{x^3}$$

$$= (x^3)^{1/3}$$

$$= x^{3 \cdot \frac{1}{3}}$$

$$= x$$