# Exercises on 2.5 Continuity

## Recall

We say a function $f$ is continuous at a point 'a' if

$$\lim_{x \to a} f(x) = f(a)$$

The following also holds:

(i) $f(a)$ is defined ✓

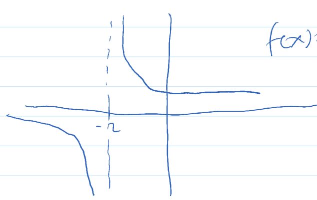
(ii) $\lim_{x \to a} f(x)$ exist ✓

(iii) $\lim_{x \to a} f(x) = f(a)$ ✓

## Exercise 1

Why is $f$ discontinuous at $a = -2$

$$f(x) = \frac{1}{x+2}$$




$f(x) = \frac{1}{x+2}$

$\boxed{f(-2) \text{ is not defined}}$

$$f(-2) = \frac{1}{-2+2} = \frac{1}{0}$$

Indeterminate

② Why is $f$ discontinuous at $a = -1$

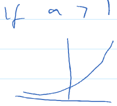
$$f(x) = \begin{cases} x+3 & \text{if } x \le -1 \\ 2^x & \text{if } x > -1 \end{cases}$$

exponential functions

$f(x) = a^x$

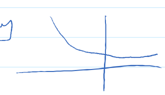
if $a > 1$

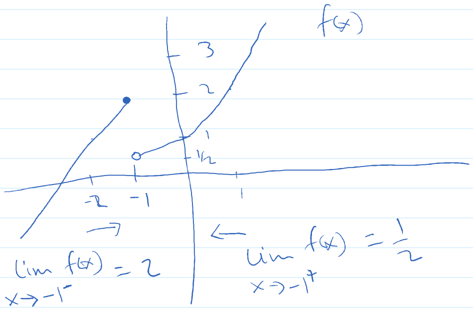
growth



if $0 < a < 1$

decay






$f(x)$

$\lim_{x \to -1^-} f(x) = 2$

$\lim_{x \to -1^+} f(x) = \frac{1}{2}$

$2^{-1} = \frac{1}{2}$

$2^0 = 1$

$2^1 = 2$

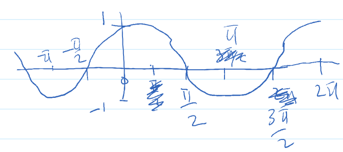
$\lim_{x \to -1} f(x)$ DNE

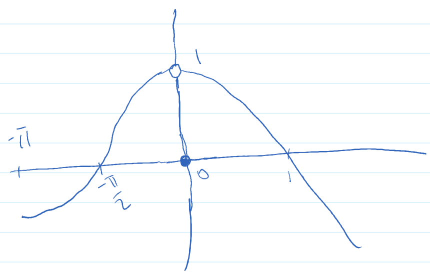
③ why is f discontinuous at $a=0$ ✓

$$f(x) = \begin{cases} \cos(x) & \text{if } x<0 \\ 0 & \text{if } x=0 \\ 1-x^2 & \text{if } x>0 \end{cases}$$

One of the goal of College Algebra and precalculus is that you recognize functions from memory

$f(x) = \cos(x)$




$f(0) = 0$ (defined at 0)

$\lim_{x\to 0} f(x) = 1$

But

$\lim_{x\to 0} f(x) \neq f(0)$

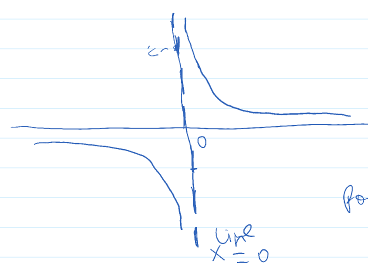
$1 \neq 0$

## 2.6 Limits at Infinity ; Horizontal Asymptotes

Recall

Vertical Asymptotes — defined at the point excluded from the domain in a Rational function

Example (i) $f(x) = \frac{1}{x}$

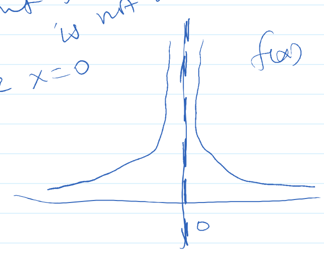



Point $x=0$ is not in the domain of f

(ii) $f(x) = \frac{1}{x^2}$

Point $x=0$ is not in the domain of f

line $x=0$




(Be sure you know the difference between Vertical asymptotes and 'holes')

Hint, $f(x) = \frac{1}{x-2}$ vs $f(x) = \frac{(x+1)(x-2)}{x-2}$

Line $x=2$ V.A — Point $x=2$ is a hole

## Horizontal Asymptotes (H.A)

$$f(x) = \frac{P(x)}{Q(x)}, \quad P, Q \text{ are polynomial functions}$$

$Q(x) \neq 0$

If deg of $P(x)$ < deg of $Q(x)$

suppose $P(x) = x^1 + 4$, deg = 1 (highest exponent on $x$)

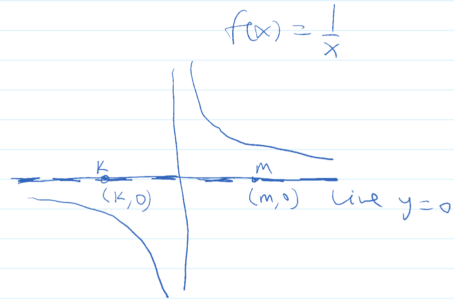
suppose $Q(x) = x^2 + 3$, deg = 2 (highest exponent on $x$)

H.A line $y = 0$

Example

$f(x) = \frac{1}{x}$ deg = 0 $(1 = x^0)$, deg = 1

H.A line $y = 0$



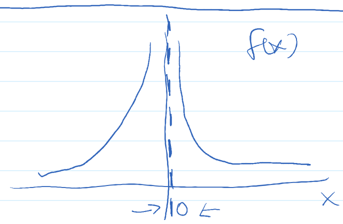

Recall in 2.2 & 2.3 — Infinite limit and vertical asymptote

(Limit DNE)

If $\lim_{x \to a} f(x) = \infty$ then line $x = a$ is a vertical asymptote

Example

$$\lim_{x \to 0} \frac{1}{x^2} = \infty$$



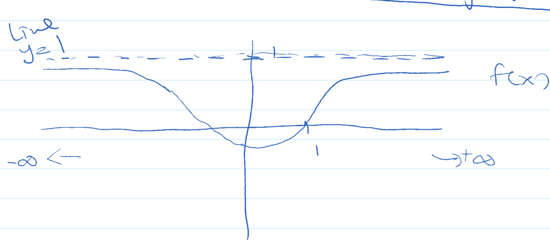

Limit at infinity and horizontal asymptotes

Today

If $\lim_{x \to \infty} f(x) = L$ then line $y = L$ is an horizontal asymptote

Example

$$\lim_{x \to \infty} \frac{x^2 - 1}{x^2 + 1} = 1$$




Definitions

suppose $f$ is a function defined on $(-\infty, a)$

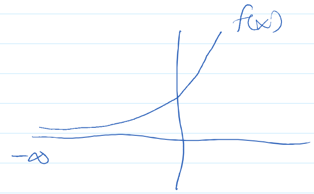
Then $\lim f(x) = L$

$x \to -\infty$

(Same as saying

$f(x) \to L$

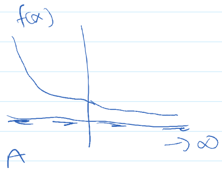
as $x \to -\infty$




$$\lim_{x \to -\infty} 2^x = 0$$

so line $y = 0$ H.A

$$\lim_{x \to \infty} \left(\frac{1}{2}\right)^x = 0$$




line $y=0$ H.A

**Theorem**

If $r > 0$ is a rational number

then $\displaystyle \lim_{x \to \infty} \frac{1}{x^r} = 0$

**Exercise**

$$\lim_{t \to -\infty} \frac{3t^2+1}{t^3-4t+1} = \lim_{t \to -\infty} \frac{\frac{3t^2+1}{t^3}}{\frac{t^3-4t+1}{t^3}}$$

$$= \lim_{t \to -\infty} \frac{\frac{3t^2}{t^3} + \frac{1}{t^3}}{\frac{t^3}{t^3} - \frac{4t}{t^3} + \frac{1}{t^3}}$$

**Theorem**

$$\boxed{\lim_{x \to \infty} \frac{1}{x^r} = 0}$$

$$= \lim_{t \to -\infty} \frac{\frac{3}{t} + \frac{1}{t^3}}{1 - \frac{4}{t^2} + \frac{1}{t^3}}$$

$$= \frac{\lim_{t \to -\infty}\left(\frac{3}{t} + \frac{1}{t^3}\right)}{\lim_{t \to -\infty}\left(1 - \frac{4}{t^2} + \frac{1}{t^3}\right)}$$

$$\boxed{\lim_{x \to \infty} \frac{1}{x^r} = 0 \quad (r > 0)}$$

$$= \frac{\lim_{t \to -\infty} \frac{3}{t} + \lim_{t \to -\infty} \frac{1}{t^3}}{\lim_{t \to -\infty} 1 - \lim_{t \to -\infty} \frac{4}{t^2} + \lim_{t \to -\infty} \frac{1}{t^3}}$$

$$= \frac{0 + 0}{1 - 0 + 0} = \frac{0}{1} = 0$$

$1 - 0 + 0$ $\quad$ $1$