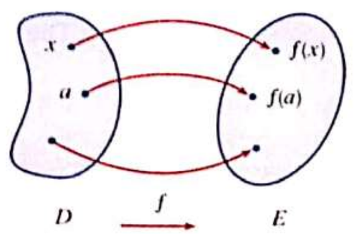
Function Review

A **function** f is a rule that assigns to each element x in a set D exactly one element, called $f(x)$, in a set E.



Examples

$\boxed{f(x) = x}$

$f(1) = 1$

$f(2) = 2$

$\vdots$

Domain: all real numbers

$(-\infty, \infty)$

$(-\infty, 0) (0, \infty)$

$-\infty \quad 0 \quad \infty$

$f(x) = x^2$

$f(1) = 1$

$f(2) = 4$

$f(3) = 9$

Domain: all real

$f(x) = \frac{1}{x}$

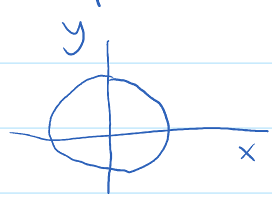
Domain: all real except 0

$(-\infty, 0) \cup (0, \infty)$

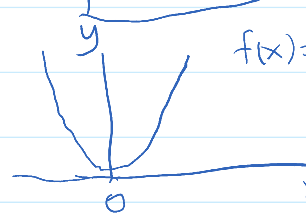
~~The Vertical Line Test~~ A curve in the xy-plane is the graph of a function of x if and only if no vertical line intersects the curve more than once.

not a function

Equation vs function



$x^2 + y^2 = 1$



$f(x) = x^2$

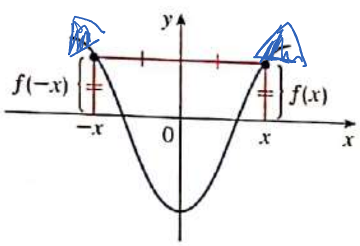


FIGURE 19
An even function

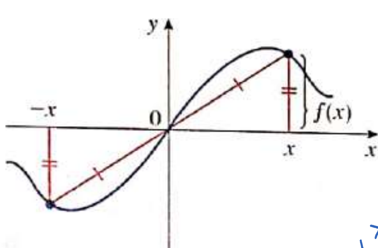


FIGURE 20
An odd function

Even functions

$f(-x) = f(x)$

Example

$f(x) = x^2$

$f(-x) = (-x)^2 = x^2$

$f(x) = x^4$

$f(x) = x^6$

odd functions

$f(-x) = -f(x)$

Example

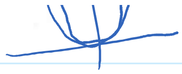
$f(x) = x^3$

$f(-x) = (-x)^3$

$= (-x) \cdot (-x) \cdot (-x)$

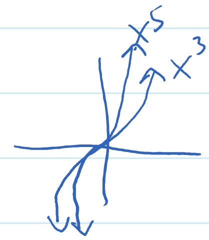
$= -x^3$

An odd function



$f(x) = x^6$

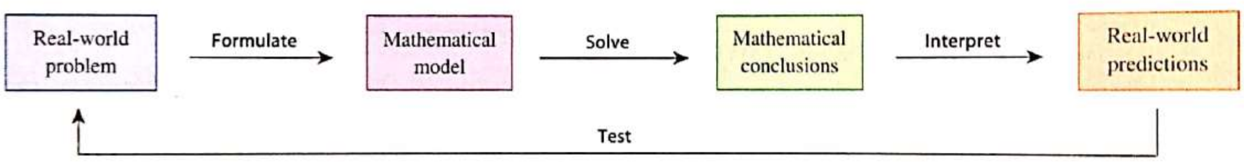
$= (-x)\cdot(-x)\cdot(-x)$
$= -x^3$
$= -f(x)$



> A function f is called **increasing** on an interval I if
>
> $$f(x_1) < f(x_2) \qquad \text{whenever } x_1 < x_2 \text{ in } I$$
>
> It is called **decreasing** on I if
>
> $$f(x_1) > f(x_2) \qquad \text{whenever } x_1 < x_2 \text{ in } I$$

Increasing on $[1,2] \cup [3,\infty)$

Decreasing on $[2,3]$



when we use functions to study real life problem, we call them mathematical models

Linear Models

When we say that y is a **linear function** of x, we mean that the graph of the function is a line, so we can use the slope-intercept form of the equation of a line to write a formula for the function as

$$y = f(x) = \boxed{mx^1} + b$$

leading term

← linear, leading term has exponent $= 1$

quadratic
$y = f(x) = ax^2 + bx + c$
$(a \neq 0)$

Polynomials

A function P is called a **polynomial** if

$$P(x) = \boxed{a_n x^n} + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$$

where n is a nonnegative integer and the numbers $a_0, a_1, a_2, \ldots, a_n$ are constants called the **coefficients** of the polynomial. The domain of any polynomial is $\mathbb{R} = (-\infty, \infty)$. If the **leading coefficient** $a_n \neq 0$, then the **degree** of the polynomial is n. For example, the function

$$P(x) = 2x^6 - x^4 + \tfrac{2}{5}x^3 + \sqrt{2}$$

is a polynomial of degree 6.

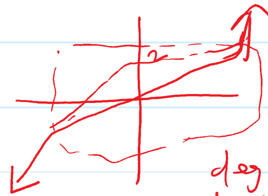
Graph
$f(x) = \boxed{x^4} + 2$
even

$f(x) = \boxed{x^5} + 2$

is a polynomial of degree 6.

$f(x) = \boxed{x^5} + 2$ ~~$x^2 + 4$~~

odd



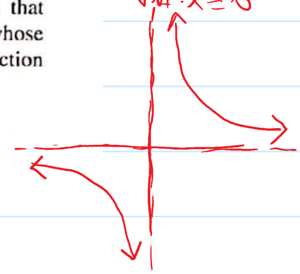
degree num = 0

~~degree~~ denom = 1

$1 = x^0$

$\boxed{f(x) = \frac{1}{x^1}}$

V.A: $x = 0$



x=-2 x=2

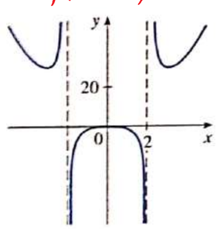


FIGURE 18

$f(x) = \dfrac{2x^4 - x^2 + 1}{x^2 - 4}$

Rational Functions

A **rational function** f is a ratio of two polynomials:

$$f(x) = \frac{P(x)}{Q(x)}$$

where P and Q are polynomials. The domain consists of all values of x such that $Q(x) \neq 0$. A simple example of a rational function is the function $f(x) = 1/x$, whose domain is $\{x \mid x \neq 0\}$; this is the reciprocal function graphed in Figure 14. The function

$$f(x) = \frac{2x^4 - x^2 + 1}{x^2 - 4}$$

is a rational function with domain $\{x \mid x \neq \pm 2\}$. Its graph is shown in Figure 18.

Dom

$(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

V.A: $x = -2$

$x = 2$

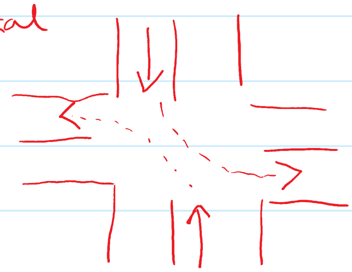
H.A: None

degree of a polynomial

the highest exponent

Horizontal asymptote

H.A: $y = 0$ $(0 < 1)$



Domain

$(-\infty, 0) \cup (0, \infty)$

Vertical asymptote line $x = 0$

Vertical and Horizontal Shifts Suppose $c > 0$. To obtain the graph of

$y = f(x) + c$, shift the graph of $y = f(x)$ a distance c units upward

$y = f(x) - c$, shift the graph of $y = f(x)$ a distance c units downward

$y = f(x - c)$, shift the graph of $y = f(x)$ a distance c units to the right

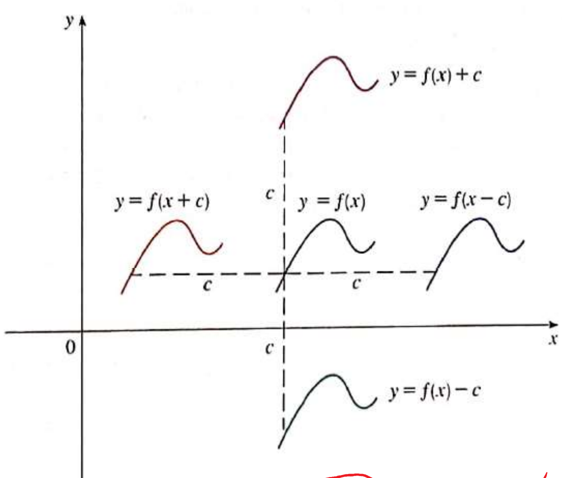
$y = f(x + c)$, shift the graph of $y = f(x)$ a distance c units to the left

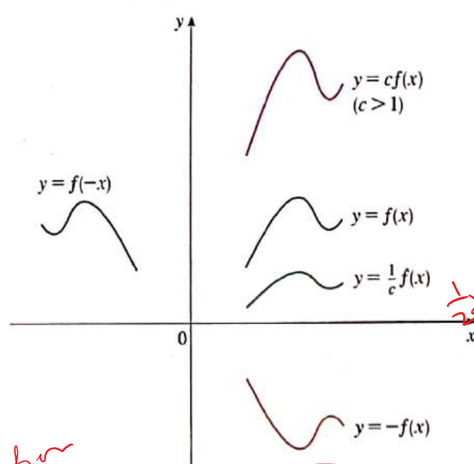
$r(x) = 2x^2$

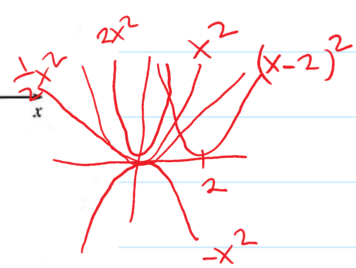
$f(x) = x^2$

$h(x) = -x^2$

$g(x) = (x-2)^2$







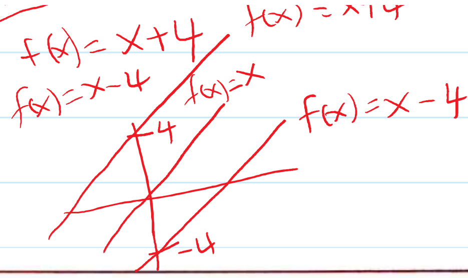
$\boxed{f(x) = x}$ linear function

$\boxed{f(x) = x^2}$

$f(x) = x + 4$ $f(x) = x + 4$

$f(x) = x - 4$ ~~$f(x) = x$~~ $f(x) = x - 4$

$g(x) = 2(x-3)^2 + 4$



$g(x) = 2(x-3)^2 + 4$

Vertical stretch

Horizontal shift right

vertical shift up

Vertical and Horizontal Stretching and Reflecting Suppose $c > 1$. To obtain the graph of

$y = cf(x)$, stretch the graph of $y = f(x)$ vertically by a factor of c

$y = (1/c)f(x)$, shrink the graph of $y = f(x)$ vertically by a factor of c

$y = f(cx)$, shrink the graph of $y = f(x)$ horizontally by a factor of c

$y = f(x/c)$, stretch the graph of $y = f(x)$ horizontally by a factor of c

$y = -f(x)$, reflect the graph of $y = f(x)$ about the x-axis

$y = f(-x)$, reflect the graph of $y = f(x)$ about the y-axis

Combinations of Functions

Two functions f and g can be combined to form new functions $f + g$, $f - g$, fg, and f/g in a manner similar to the way we add, subtract, multiply, and divide real numbers.

Definition Given two functions f and g, the **sum**, **difference**, **product**, and **quotient** functions are defined by

$$(f + g)(x) = f(x) + g(x) \qquad (f - g)(x) = f(x) - g(x)$$

$$(fg)(x) = f(x)\,g(x) \qquad \left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

Definition Given two functions f and g, the **composite function** $f \circ g$ (also called the **composition** of f and g) is defined by

$$(f \circ g)(x) = f(g(x))$$

1 Definition A function f is called a **one-to-one function** if it never takes on the same value twice; that is,

$$f(x_1) \neq f(x_2) \qquad \text{whenever } x_1 \neq x_2$$

Horizontal Line Test A function is one-to-one if and only if no horizontal line intersects its graph more than once.

2 Definition Let f be a one-to-one function with domain A and range B. Then its **inverse function** f^{-1} has domain B and range A and is defined by

$$f^{-1}(y) = x \iff f(x) = y$$

for any y in B.

domain of f^{-1} = range of f

range of f^{-1} = domain of f

$f^{-1}(x)$ does *not* mean $\dfrac{1}{f(x)}$